

Mathletics

Series F & G

# Problem Solving

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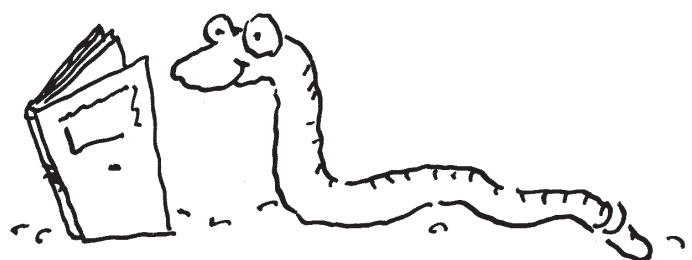
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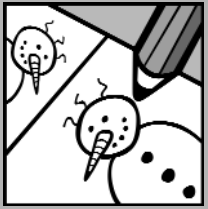
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# Problem Solving – Series F & G

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# Draw a diagram

## Rationale

Diagrams are powerful tools for setting out and showing clear understanding of various problems and for planning their solutions. Diagrams help us understand the connections and interactions between parts of a problem. When we have this information, we can understand processes, moves and plans more fully.

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## Teaching *Draw a diagram*

The following considerations are necessary when teaching *Draw a diagram*.

- A Types of diagrams and their best uses
- B Main skills

### A Types of diagrams used

- 1 **Tree diagrams** – useful when planning combinations of items. Family trees belong in this category as well. They show connections between family members.
- 2 **Timelines** – used when the history of events needs organising so that relationships can be seen.
- 3 **Pictures** – used to show how spaces can be organised.
- 4 **Venn diagrams** – used when data needs to be organised or sorted into groups and common items found.
- 5 **Fraction bars or circles** – used to show processes with fractions of a whole. Percentages can be worked with similar diagrams.

### B 4 main skills

#### I Utilise the given diagram

Each page gives practice at one kind of diagram. Students should review the type of problem and the method of diagramming. Ask them to give similar problems orally and check that their problems fit the type of diagrammatic solution being studied.

#### 2 Convert data to a visual format

Students should use plenty of space and leave the diagram in place for marking and sharing. Acknowledgement should be made for a good attempt at drawing the diagram, to encourage students to place importance on the process as well as the answer.

#### 3 Checking the solution

Re-read the problem and check that the diagram is a valid *picture* of the data. Be sure that full understanding is in place.

#### 4 Explain the solution

The ability to verbalise the solution and answer questions about it demonstrates good mathematical understanding. eg Why did you choose that diagram? How did you show the money left to be spent?



**Worksheet 1**

**HANDY TREES**

Students are given the structure of the tree diagram. They only need to put in labels and answer questions from their conclusions, which they should be encouraged to verbalise. The first tree diagram is set out vertically, while the second is set out horizontally. Discuss the use of the two formats. In Problem #2, students put the names of the jobs on the branches and the names of the children on the answer lines.

**Worksheet 3**

**LINES OF HISTORY**

Turn the page sideways to record the life events. Make each record a shortened note of the narrative, eg Began school. Discuss what other conclusions you can draw from the completed timeline.

**Worksheet 5**

**LET'S SORT IT OUT**

Venn diagrams are taught with an example. Explain how overlapping areas show the data which belongs in two sections. Assist by giving the names to the sections before the students begin. Use crosses or dots for people.

**Worksheet 7**

**JOLLY JETSONS' JOY**

Make sure students understand the way *one* (1) is shown in diagrammatic form — either a rectangle, square or circle. This is then divided according to the fractions being shown. Accuracy is most important — sloppy work will make it hard to see the solution clearly. Check with the original problem to be sure that the story and the diagram match.

**Worksheet 2**

**FAMILY TREE**

The first family tree demonstrates how to set this out. In the second, the students are to use the first as a template. Discuss how the diagram matches the narrative. Teach how the children of a marriage are shown on the branch which comes from a married couple.

**Worksheet 4**

**TRY THIS ON YOUR FRIENDS**

This set of exercise encourages visual planning as well as making a guess and checking from the drawing. Discourage students from telling others how the solution is found so that they all try for themselves.

**Worksheet 6**

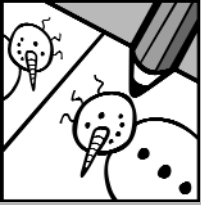
**SORTING USING DIAGRAMS**

Students are expected to work these Venn diagrams for themselves, beginning with naming the sections, then placing crosses for the correct number in each category. Some trial and error is required. Make sure the students check with the original information in the narrative — most necessary.

**Worksheet 8**

**THERE'S A FRACTION TOO MUCH FRICTION**

The first two problems here use fractions of more than *one*. Make sure students understand how this is shown. Check and discuss.



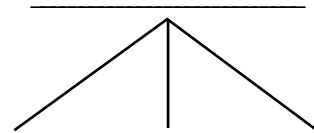
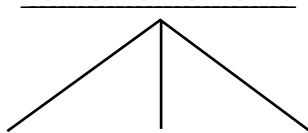
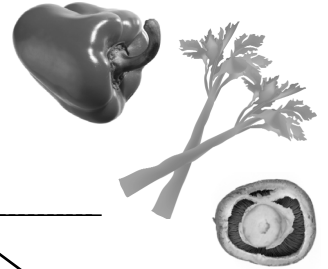
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# Handy trees

**Use tree diagrams to solve the following.**

1 Henry's Pizza Pantry sells pizzas with a variety of toppings, if you like vegetables! The menu includes red peppers, mushrooms and celery for extra healthy pizzas. You can have cheese with one vegetable or cheese and ham with one vegetable.



What are the different pizzas that Henry makes?

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2 The scouts have lots of fund raising jobs to do. The biggest job for everybody will be a massive weeding of the gardens. After that, there is sweeping, mowing and digging to be done. Finally the groups will have to divide into 6 to do planting, staking, watering, mulching, trimming and carting away rubbish.

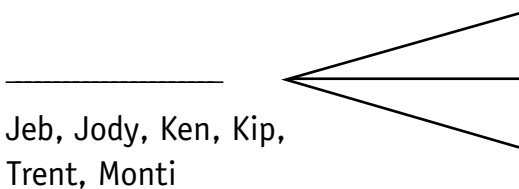


There is mass confusion at first, but the scout leader brings order by drawing a tree diagram. What would each person do for each job? Write the jobs on the branches of the trees.

**First Job**

**Second Job**

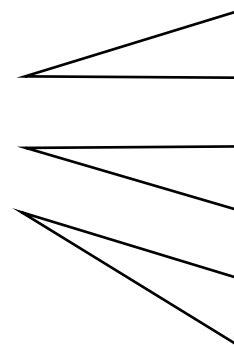
**Third Job**



\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



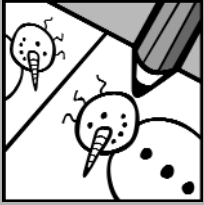
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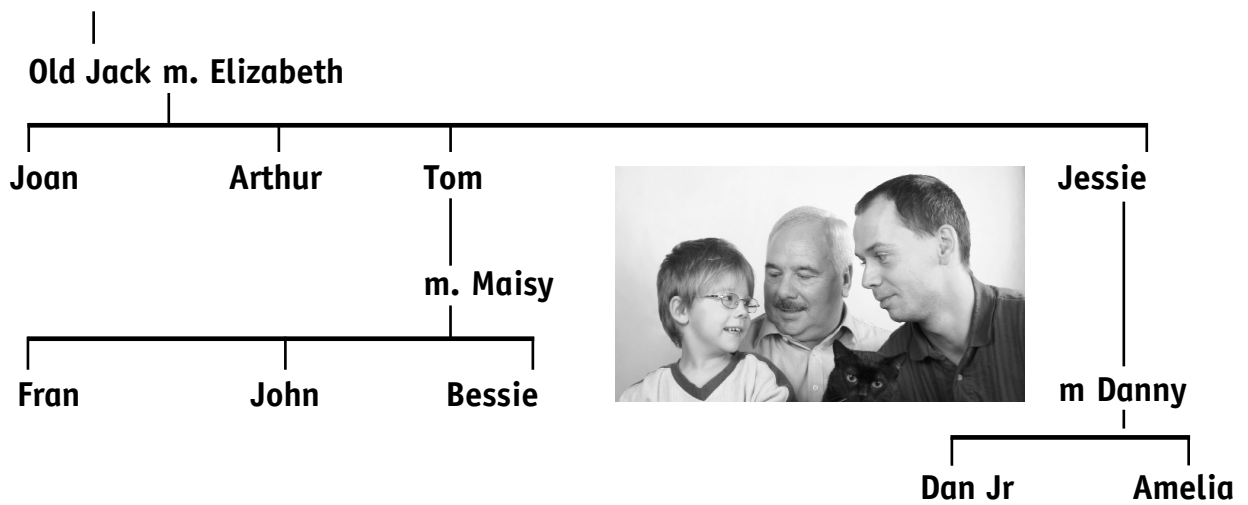
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# Family tree

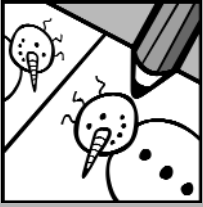
**A family tree shows a family's relationships and numbers of family members in each generation. It has branches like an upside down tree.**

1 Old Jack had a wife, Elizabeth, and 4 children. Two of his 4 children married; the other two, Joan and Arthur, remained single. Tom and his wife Maisy, had 3 children, Fran, John and Bessie, while Jessie and her husband Danny, had 2 children, Dan Jr and Amelia. How many people came to their Christmas dinner (when the whole family always met)? \_\_\_\_\_

Circle all of the youngest generation on this family tree.



2 The first settlers to arrive at Worthington were Martin and his wife Myrtle. Their children were Stanley, Gregory and Maude. Stanley's wife was Vivienne and they had 3 children, Harry, Stan Jr and Felicity. Gregory remained single. Maude married Peter and their children were Zachary, Joseph and Larry. Draw the family tree to show the three generations of this family.



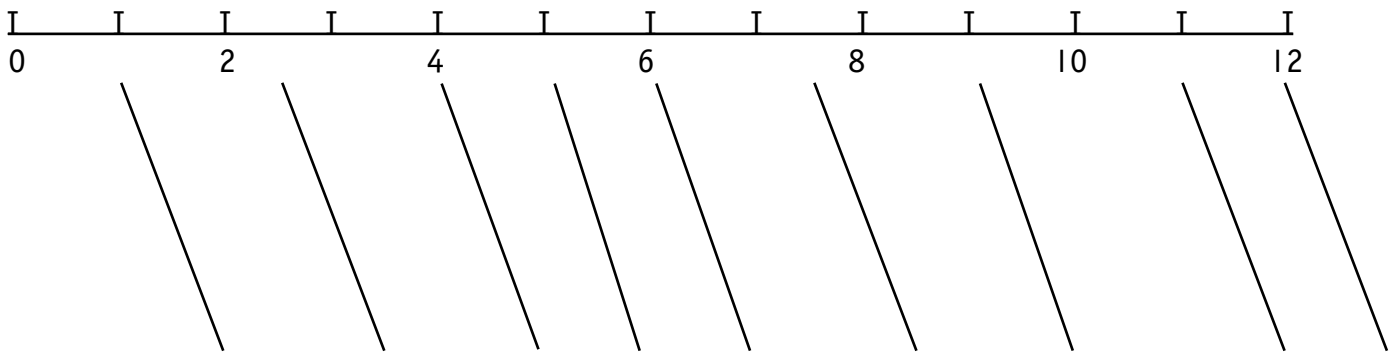
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## Lines of history

**1 a** Place the following important events on the timeline of part of Eva's life.

At 1 year of age, her first tooth appeared. At  $2\frac{1}{2}$  years, she attended nursery school. When she was 4, she went to preschool, before beginning school at 5 years. The day she turned 6, her Dad bought her a bike and when she was  $7\frac{1}{2}$  she broke her arm. On her 9th birthday, she had a sleepover with friends for the first time. At 11 she entered Year 6. At the end of that year she graduated to secondary school.



**b** There were more important events in Eva's life before she began school than after.  
true/false \_\_\_\_\_

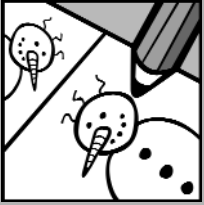
**2 a** Draw a timeline to plan out the year on Grant's Dairy. Divide it into 12 months.

In June, 12 new calves were born while in May, 3 cows had been sold. Also in May, there had been a blockage in the water pipes to the cows' feeding yard. At the beginning of spring, another 12 cows had calves and in October, 5 new cows were bought for the herd. In February, a new dairy worker had to be trained and then in November another one had to be trained. During March the hay was cut for winter feed.



**b** The busiest season of the year was spring. true/false \_\_\_\_\_



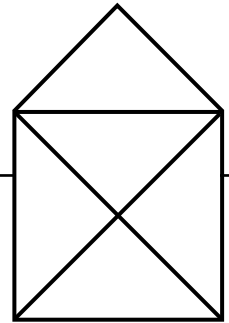


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## Try this on your friends

**Study this drawing for a minute. The drawings you are to make are based on this shape. Do not take your pencil from the paper in these exercises.**

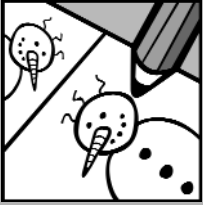


**1** Make 2 triangles by drawing 5 straight lines.

**2** Make 3 triangles by drawing 7 straight lines.

**3** Make 4 triangles by drawing 7 straight lines.

**4** Make 5 triangles (the complete shape) using 8 lines, without taking your pencil from the page.



Name \_\_\_\_\_

Date \_\_\_\_\_

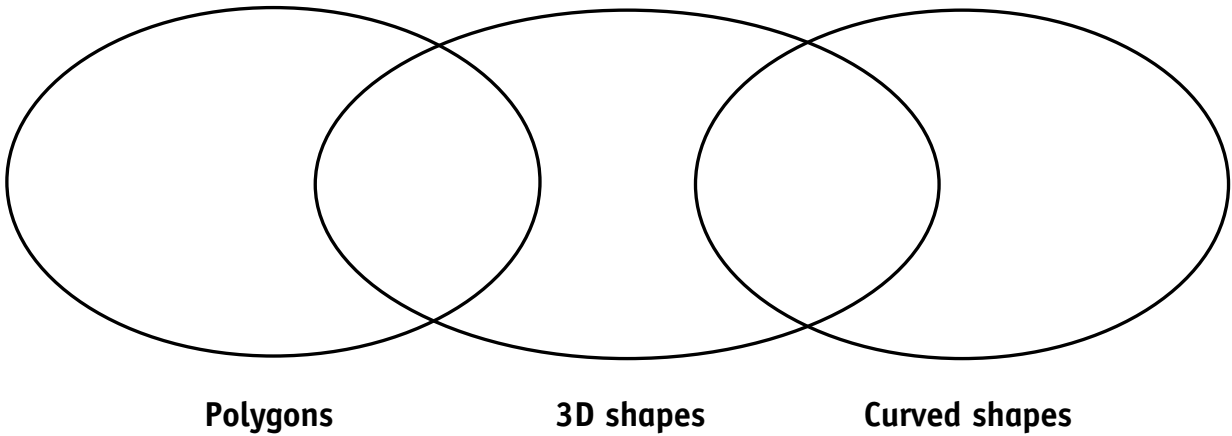
# Let's sort it out

**Use Venn diagrams to help organise data into groups.**

<p>Example</p> <p>8 children went swimming, then had ice-cream.</p> <p>6 played soccer then had ice-cream.</p> <p>4 children had only ice-cream.</p>	
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1 Put the shapes in the correct sections to show their properties.

square, circle, rectangle, ellipse, cube, hexagon, pyramid, octagon, triangle, cylinder, cone

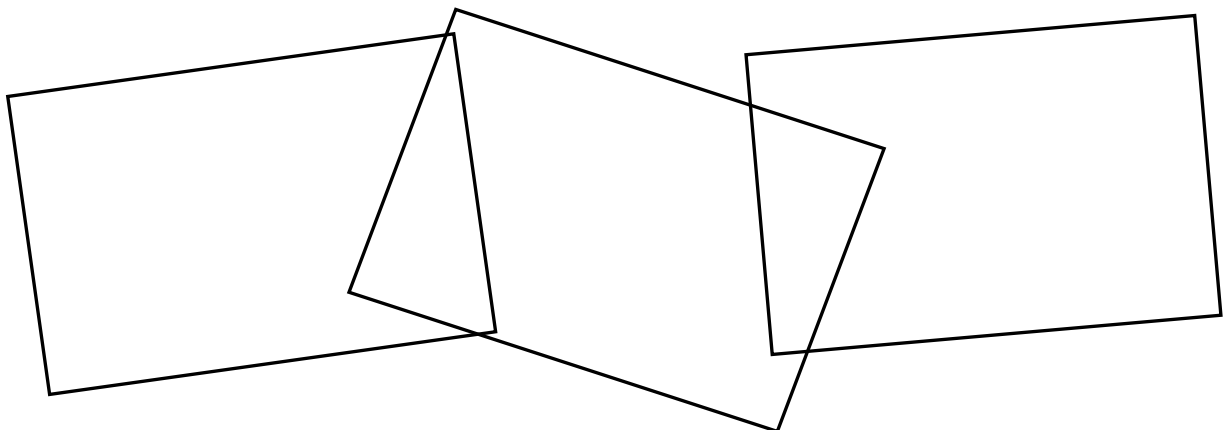


2 If the following numbers were making teams, they'd line up in these ways:

Prime numbers, Triangular numbers, Multiples of 7.

a Put these numbers in their correct teams. 3, 10, 11, 14, 15, 19, 28, 35

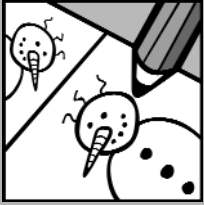
b Which numbers are members of more than one team? \_\_\_\_\_



Prime numbers here!

We are triangular numbers.

Multiples of 7 over here!

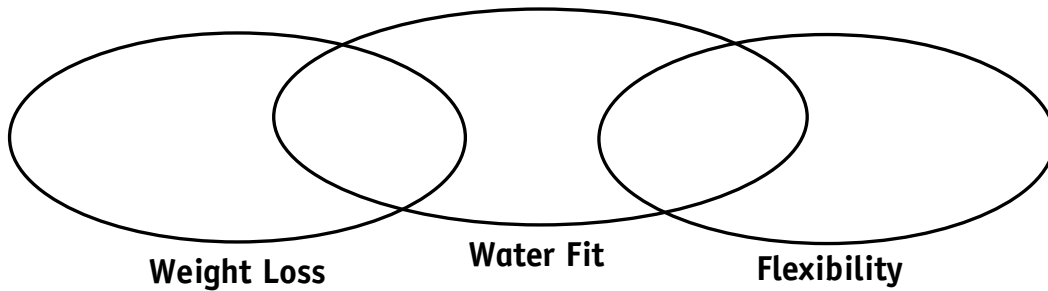


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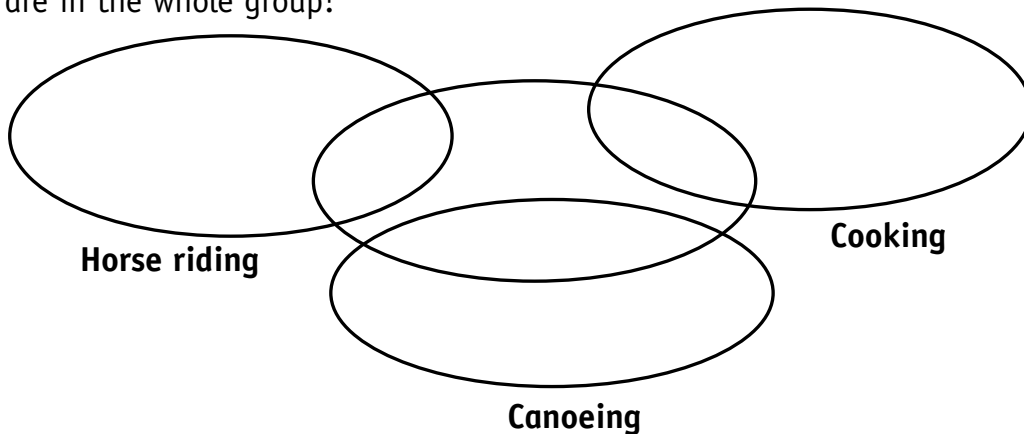
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# Sorting using diagrams

1 At the Finer Fitness Club, 22 members were in the Weight Loss Plan and 24 were in the Flexibility Plan. 8 members were in the Flexibility Plan as well as the Water Fit Program. 6 were in the Weight Loss Plan as well as in the Water Fit Program. The Weight Loss, Flexibility and Water Fit Programs each had 16 people who were in no other Plan or Program. How many people were in the whole club?

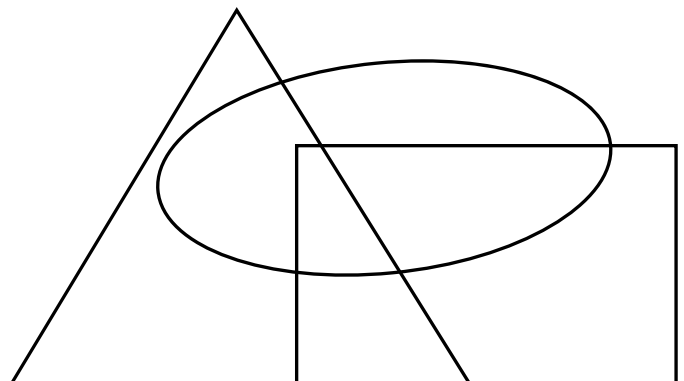


2 When the Get-It-Done-Girls took off for their Camp during the summer, they were excited about the new activities they would try. 6 would try horse riding, 8 would try canoeing and 6 would try cooking, all for the first time. In the whole group, 9 girls were doing an activity a second time; 4 had done horse riding before, 3 had done cooking and 2 had done canoeing. How many girls are in the whole group?



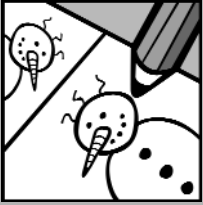
3 a Put the following numbers in their correct areas — those divisible by 5 in the triangle; those divisible by 6 in the ellipse; those divisible by 4 in the rectangle.

- 5, 15, 18, 24, 28,  
30, 32, 40, 54, 75



b Which numbers are divisible by 5 and 6? \_\_\_\_\_

c Which numbers are divisible by 4 and 6? \_\_\_\_\_



Name \_\_\_\_\_

Date \_\_\_\_\_

# Jolly Jetsons' joy

- 1 a It rained most of the weekend when the Jetsons went on holiday. Three-quarters of the rain fell on Saturday. Colour this fraction blue. Half of that fell on Saturday morning. Draw black lines over this fraction.



- b More rain fell on Saturday morning than during the rest of the weekend. true/false \_\_\_\_\_
- 2 a Grandpa Jetson gave one-half of his money to his children. Colour this section red. He then gave one-quarter of what was left to his grandchildren. Colour this blue. Half of the rest went to a charity. Colour this yellow.



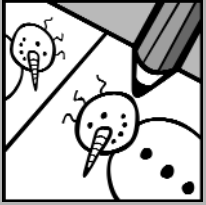
- b Grandpa gave the smallest amount to his grandchildren. true/false \_\_\_\_\_
- 3 a The Jetsons spent four-fifths of their money on accommodation. One-quarter of this was for parents' rooms, and one-third of the remaining accommodation money was for children's rooms. Colour this on the diagram.



- b The parents' rooms cost more than the children's rooms. true/false \_\_\_\_\_
- 4 a A birthday cake for Jimbo Jetson was cut in half. One half was cut into 8 for the Little Jetsons, while the other half was cut into 6 for the Big Jetsons. 2 pieces of the Little Jetsons' cake were left and 2 pieces of the Big Jetsons' cake were left. Colour the diagram to show this.



- b The Big Jetsons left more cake than the Little Jetsons. true/false \_\_\_\_\_



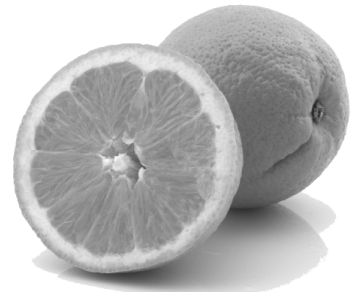
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# There's a fraction too much friction

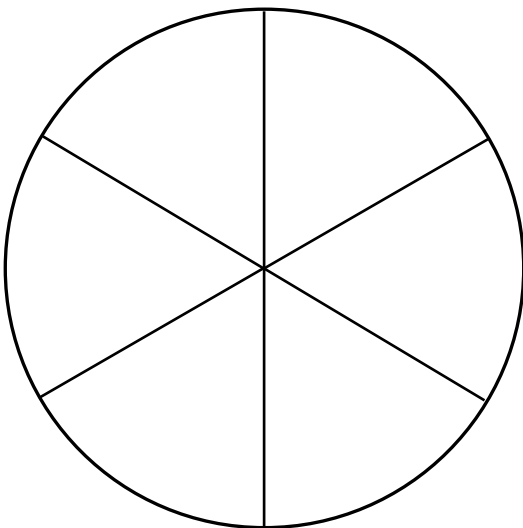
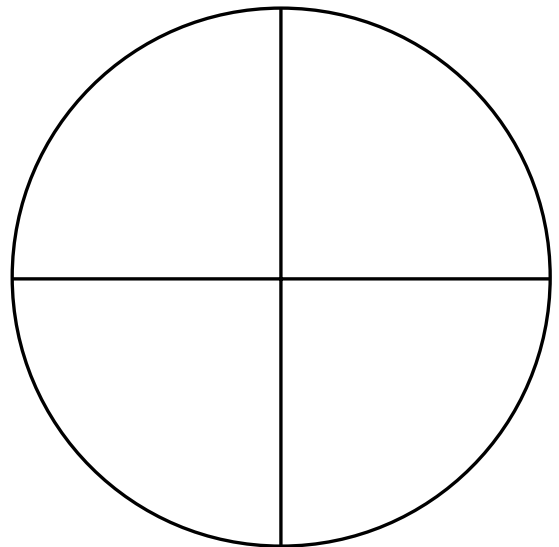
## Draw diagrams for each question.

- 1 Four children each had two-thirds of an orange at half time in the tennis game and there were four-thirds of orange left.  
How many oranges were there to start? \_\_\_\_\_



- 2 From two and a half metres of ribbon we cut eight pieces, each one-fifth of a metre in length.  
How much ribbon is left? \_\_\_\_\_

- 3 At the District Show I intend to spend one-quarter of my money on food, three-eighths of my money on Show Bags and I hope to still have three-eighths of my money for rides.  
If I want £45 for rides, how much money will I need to start with? \_\_\_\_\_



- 4 In the village of Gophourit, most children spend one-third of the day sleeping, one-quarter of the day at school and just one-twelfth eating.  
How much time is left for other activities?  
\_\_\_\_\_



# Look for patterns

## Rationale

Discovering the patterns in groups of numbers can be fun, apart from being very useful in solving problems. Teaching the recognition of patterns and the application of rules lays a foundation for teaching algebra. The discovery of patterns in number makes mathematical relationships more interesting and engaging for young students. The ability to manipulate numbers leads to strong understanding of mathematical thinking.

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## Teaching *Look for patterns*

Patterns may be in an arrangement of elements or in the form of a sequence. Both have rules which we must learn to recognise. The following points should be considered in teaching problem solving using pattern recognition as a strategy.

**A** Types of patterns

**B** Main skills

### A Types of patterns

Discuss and record the types of patterns and where they are found. Shapes, sizes, numbers, letters, positions and colours can all form the elements of patterns.

Discuss how the use of patterns is of benefit, eg progression of house numbers, number plates on cars, traffic lights. Build a data bank of such patterns in our lives.

### B Main skills

**1** **Recognising what is a pattern** and what is not, by recognising common elements. There are examples all around us in visual contexts. Also explore musical patterns of sound and rhythm.

**2** **Describing a pattern** eg three diamonds are followed by 2 ellipses. Accurate descriptors are essential.

**a** Use words to explain the pattern.

**b** Write them down so others can understand.

**c** Use position words and ordinal numbers to assist explanation.

**3** **Complete a pattern** given by someone else.

Recognise the pattern, check it and continue it as required.

**4** **Give a rule for a pattern**

If possible, express this in numerical terms, eg *it goes up by 7* should be expressed as *the number plus 7*.

**5** **Use the recognition of rules in solving problems**

Learn to detect where a pattern has been formed in the information given. Use this knowledge to solve the problem.



## Worksheet 1

### TRAIN YOUR EYE

Students study the pattern and then discuss how the mirror image will appear from the position of the mirror. They will be helped by realising that the shape nearest the mirror will be the one that appears first in the mirror reverse.

## Worksheet 3

### NUMBERS HIDING

There is some *trial and error* in these solutions. Initially, there will be a guess as to how the series progresses in order to find the numbers hiding under the Xs.

In the Input/Output section, a study of the relationship between the Input and Output columns will show how the pattern is formed and therefore how it can be continued.

## Worksheet 5

### NUMBER PLATES

Students need to recognise where a series is leading and what its parts may be. Checking is very necessary. They can explain their reasons for their solutions.

## Worksheet 7

### WORK TO RULE

These patterns build on the table format of producing a pattern from a given original number or term. Discussion should include how, if we have a rule, we can determine an answer. eg If  $N = 6$ , what is  $N \times 2 \div 3$ ? Calculation originates from the top of the table which is N.

## Worksheet 2

### SEQUENCES

Sequences are built by ordering a set of numbers, words or objects, which have a common descriptor, eg multiples of 5. The sequence has a rule for its progression. By studying this progression and recognising common elements, it can be extended or checked for accuracy.

## Worksheet 4

### OLYMPIC CHAMPS

Students need to read instructions carefully. Checking the solution against the information is most important. Stress that students must check, to be sure that their answer makes sense, in terms of the question.

## Worksheet 6

### SERIES WITH RULES

Read the information carefully. After following instructions to form patterns, students are asked to comment on patterns seen forming in Q4. Why might this be?

## Worksheet 8

### PAYING FOR CREDIT

Discussion can begin with how easy it is to accumulate credit with cards and with a mobile phone. Then, discuss how we must pay all the money back and that charges change according to how long we owe it. Each problem is phrased differently and requires different operations to arrive at solutions.

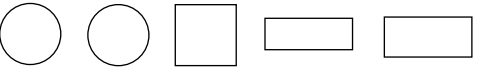


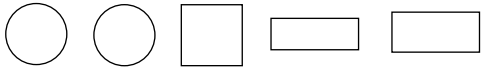
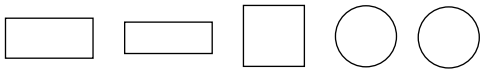
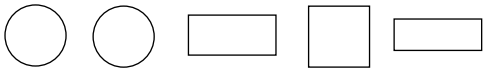
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# Train your eye

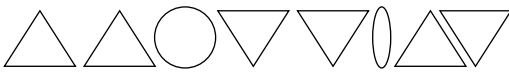
Which design on the right is the *mirror image* of the one on the left?

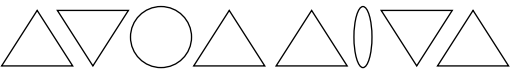
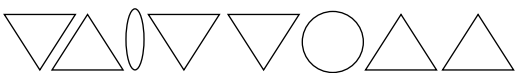

1  mirror

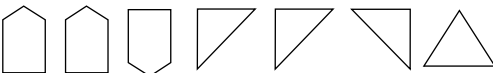
- a 
- b 
- c 




2  mirror

- a 
- b 
- c 

3 mirror 

- a 
- b 
- c 

4 mirror 

- a 
- b 
- c 

5 Draw the mirror image.

y y z z 9 d s s

mirror





Name \_\_\_\_\_

Date \_\_\_\_\_

# Sequences

**A set of numbers following a pattern is called a *sequence*.**

**We can make a sequence beginning and ending anywhere.**

- 1 Make a sequence of 8 numbers according to each given rule.
  - a multiples of three; starting from 3 \_\_\_\_\_
  - b divide by 10; starting from 14 000 \_\_\_\_\_
  - c add 1, multiply by 2; starting from 3 \_\_\_\_\_
  - d one less than double the number; starting from 4 \_\_\_\_\_
  - e the sum of the preceding two numbers; starting from 2, 3 \_\_\_\_\_
- 2 Write the sequence of days of the week containing the letter 's'.  
\_\_\_\_\_
- 3 Write the sequence of months of the year containing the letter 'r'.  
\_\_\_\_\_
- 4 Write the sequence of prime numbers less than 40.  
\_\_\_\_\_
- 5 Write the sequence of composite numbers between 25 and 40.  
\_\_\_\_\_
- 6 Write the sequence of numbers divisible by 4 which are  $< 30$  but  $> 10$ .  
\_\_\_\_\_
- 7 Write the sequence of odd numbers which are prime and also less than 10.  
\_\_\_\_\_
- 8 Write the sequence of numbers which grows by 5, beginning with 22 and ending with 57.  
\_\_\_\_\_
- 9 How many numbers are in the sequence of multiples of 3 between 40 and 50? \_\_\_\_\_
- 10 How many numbers are in the sequence of square numbers less than 50? \_\_\_\_\_



Name \_\_\_\_\_

Date \_\_\_\_\_

# Numbers hiding

1 What numbers are hidden under the squares marked with an X?

**a**

1				X					28
	X	37							
							X		

**b**

1				17					37
		X					X		
				X					

**c**

									X
				X				32.5	
		X							16
14.5		X							1

2 Complete the magic squares.

**a**

8	1	
3		
4		2

**b**

2		6
	5	
4		8

3 Write each rule. Then fill in the blank spaces.

**a**

Input	Output
1	5
2	8
3	
	14
	17
7	

**b**

Input	Output
1	1
2	5
3	9
4	13
	21
8	
	37

**c**

Input	Output
1	2
4	17
6	37
7	
	65
15	
	401



Name \_\_\_\_\_

Date \_\_\_\_\_

# Olympic champs

**The competitors are trying everything to bring home even more medals than they won last Olympics. See how much they are training to be fitter and faster!**

- 1 The rowers trained for the 500 m Eights and think that they can improve by taking fewer strokes every time they do the 500 m. If they can cut one extra stroke from their count every week, in what week will they be doing less than 40 strokes? \_\_\_\_\_

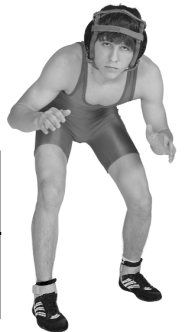


<b>Week</b>	1	2	3	4	5	6	7
<b>Strokes</b>	52	51					

- 2 Mack MacMaster was very clever at getting Frannie Fisch to improve her swim times a little each week. The following table shows how her best times progressed in a pattern. Complete the table to show her time in Week 8.

<b>Week</b>	1	2	3	4	5	6	7	8	9
<b>Time</b>	45.2s	42s	40.4s	39.6s					38.825s

- 3 Andie Artache wished to gain weight for his wrestling competition. The following chart shows his weight gains during the months leading up to the competition. Follow the pattern to complete the table.



<b>Date</b>	Mar 7	Mar 21	Apr 7	Apr 21	May 7	May 21	Jun 7	Jun 21
<b>Weight</b>	84 kg	90 kg	95 kg	99 kg				

- 4 Every 4 days, Hubie Huggers needs to improve the distance he runs in 55 sec by 0.5 metres more than he improved in the previous 4 days. On Mar 3 he can run 400 m and on Mar 7 he can run 400.5 m.

a Complete the table.

b How far will he run in 55 seconds at the end of March? \_\_\_\_\_

<b>Date</b>	Mar 3							
<b>Distance</b>	400 m							



Name \_\_\_\_\_

Date \_\_\_\_\_

# Number plates

1 Write the number plates that were made between the following:

a ABC 994 \_\_\_\_\_ ABD 001

\_\_\_\_\_

b DJ 95 AA \_\_\_\_\_ DJ 00 AB

\_\_\_\_\_

c AY 9989 \_\_\_\_\_

\_\_\_\_\_ AZ 0002

d TZ 997 \_\_\_\_\_ UA 003

e 36 JJ 49 \_\_\_\_\_ 36 JP 49

\_\_\_\_\_



2 In the town of Nockley, house numbers have an unusual way of progressing. Starting at A0 in the East or South on one side of the road, and A1 on the other, they progress by two before changing the initial letter. So A8 would be next door to B0, and A9 would be next door to B1 on the other side of the road.

What are the house numbers on either side of the following?

a \_\_\_\_\_ D8 \_\_\_\_\_

b \_\_\_\_\_ R0 \_\_\_\_\_

c \_\_\_\_\_ S1 \_\_\_\_\_

d \_\_\_\_\_ J4 \_\_\_\_\_

e \_\_\_\_\_ X2 \_\_\_\_\_

f \_\_\_\_\_ Y9 \_\_\_\_\_



3 Write the rule for progressing each of these series.

a 45, 56, 68, 81, ... Rule: \_\_\_\_\_

b 3, 8, 15, 24, 35, ... Rule: \_\_\_\_\_

c 1, 8, 27, 64, ... Rule: \_\_\_\_\_



Name \_\_\_\_\_

Date \_\_\_\_\_

# Series with rules

- 1 The Council of Moggy has to employ more cats to carry out big projects in the new year. They decide to double the number of workers plus one extra each week. Complete this table to show how many cats are employed each week in total.

Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10
1	3	7							

- 2 The Moggy workers are not very honest cats, and their tools keep disappearing. In week 1, there were 54 tools. In week 2, ten tools were lost. But each week, one less tool is lost. Complete the table to show when they will have no tools left at all.



Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10

- 3 Because of the lost tools, the working cats of Moggy are paid a little less each week. Study this table and decide how their boss works out how to pay them.

Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10
£285	£280	£276	£271	£267	£262	£258	£253	£249	£244

The cats would be paid \_\_\_\_\_  
 \_\_\_\_\_



- 4 Each day, the cats are arranged in teams which number between 3 and 8. Study the table in Question 1 and complete this table. Discuss the patterns formed.

	W1	W2	W3	W4	W5	W6	W7	W8
a number left out when in teams of 3								
b number left out when in teams of 4								
c number left out when in teams of 5								
d number left out when in teams of 6								
e number left out when in teams of 7								
f number left out when in teams of 8								



Name \_\_\_\_\_

Date \_\_\_\_\_

## Work to rule

- 1 Study the rule for changing the *Number* row.
- a Circle the numbers which are not correct, according to the rule.
- b For e – j, write their rules in algebraic form.

Number	1	2	3	4	5	9	10	15
a increase by 3; $N + 3$	4	5	6	8	8	13	13	17
b double; $N \times 2$	2	4	6	9	10	16	20	31
c squared; $N^2$	1	4	9	14	25	87	100	235
d double and add 1; $N \times 2 + 1$	3	5	7	9	11	18	21	32
e multiply by 3 and add 2; _____	5	7	10	14	16	29	32	46
f halve and add one; _____	1.5	1	2.5	3	4	5.5	6	8
g subtract 0.5; _____	0.5	1.5	2.5	3.5	5.5	8.5	10.5	15.5
h multiply by 9; _____	1	18	27	36	47	81	99	135
i multiply by 2 and subtract 2; _____	0	2	4	6	9	14	18	30
j adding consecutive odd numbers _____	2	5	10	12	14	21	23	30

- 2 Work out the rule used to change Row A each time.

Write it in the first column in algebraic form, eg  $N \times 4$ .

Row A	1	2	3	4	5	8	10	16
a	0	3	8	15	24	63	99	255
b	0.5	1	1.5	2	2.5	4	5	8
c	1	8	27	64	125	512	1000	4096
d	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	2	2.5	4
e	4	7	10	13	16	25	31	49



Name \_\_\_\_\_

Date \_\_\_\_\_

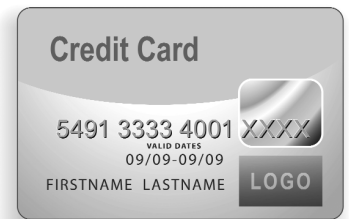
# Paying for credit

1 Paying off a credit card is a difficult but wise thing to do. Jeremy paid off an initial £100 in May, and then each month wanted to pay £50 more than the month before.

Complete the table. Be careful – all the months are not consecutive!

<b>Month</b>	May	June	Jul	Aug	Sept	Oct	Dec	Mar
<b>Total Amt Paid</b>	£100							

2 Jenny pays her credit card off differently. She owes £4 750. Her rule is to pay £50 in May then double the payment each month.



a Complete the table to help her keep a record of her expected balance each month.

b When will her balance be nil? \_\_\_\_\_

<b>Month</b>	May	June	Jul	Aug	Sept	Oct	Nov	Dec
<b>Balance</b>								

3 Poppie's credit card has interest added every month. She vows to pay £100 each month. Although she pays this amount, and it is taken from the total, every month £5 interest is being added.

a With an initial balance of £1 500, what will be her balance after 8 months?

\_\_\_\_\_

<b>Month</b>	May	June	Jul	Aug	Sept	Oct	Dec	Mar
<b>Balance</b>								

b How much will Poppie still have to pay after 15 months of paying her bill?

\_\_\_\_\_

4 Toni bought a car on credit. The car cost £7 800 and interest payments of £42 are added each month. He wants to pay it off in 12 months. How much will his monthly payments be?

\_\_\_\_\_





# Make a list

## Rationale

Setting out information in an orderly fashion, guides students to think and work systematically. When a list is made, a pattern will often emerge and the solution may come more easily if this is pointed out. Making a list is therefore a step towards finding a pattern to solve similar problems in the future.

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## Teaching *Make a list*

The following should be noted when teaching this strategy.

**A** When to use *Make a list*

**B** Main skills

### A When to use *Make a list*

Whenever a strategy is not obvious, making a list is a way to see what the information is saying. Also, when all possibilities need to be recorded and counted we make a list. Working systematically is necessary so that all possibilities can be seen to be covered.

### B Main skills

#### I Working systematically

Students must be able to decide on a starting point, then work systematically through each item, exhausting all possibilities for that item before moving on to another piece of information. eg How many three-digit numbers can be made with the digits 345? Start with the 3 in hundreds position, then 45, followed by 54. Then use 4 in the hundreds, followed by 35, then 53. Lastly, begin with the 5 in the hundreds place, followed by the 43, then 34.

#### 2 Physically setting out the list

Students will realise that they need space for the list to expand down the page. In some situations, extra paper may need to be supplied. Teachers should introduce scaffolds if required.

#### 3 Visualising/estimating

Students need to visualise possibilities, then they will have a good idea when they have covered them all.

#### 4 Recognising repeat combinations

Students must recognise that there will be repeats, which are not included, unless the problem requires it. eg A with B is the same as B with A, except where order of the items makes a difference in the combination and should therefore be included.

#### Lists become tables

Lists may be made into tables, especially where advanced students are ready to recognise this step. A table has more than one column and a heading for each column.





**Worksheet 1**

**PALINDROMES**

Palindromes are numbers or words which can be read the same backwards and forwards. Encourage working systematically, rather than jumping around from one solution to another. Decide on a starting point and work through all possibilities before moving on.

**Worksheet 2**

**AGING FAMILIES**

Scaffolds are presented for these lists. Make sure students write down the beginning facts and each progressing fact thereafter. This will help show the progress of the years and the relationship between the ages as the years progress.

**Worksheet 3**

**PAYING THE TOLL**

Lists are merged with tables so that students see how best to organise their working. Make sure students read all information carefully.

**Worksheet 4**

**MARKETING MADNESS**

Students carefully work through the information given and record it fully — they do not jump to conclusions. Remind them that full understanding of working mathematically involves careful notation of possibilities.

**Worksheet 5**

**FITNESS FANATICS**

This organisation is helpful in everyday life — it shows how rounds of competition are arranged. Point out the activities that students may become involved in.

**Worksheet 6**

**TIMETABLES**

These activities present problems involving everyday activities in which the students may become involved. Reading the information carefully is most necessary so that all points are included.

**Worksheet 7**

**FOR THE TUMMY**

Including all possibilities for variety is a useful skill. Students need to see how a few items will increase the number of possibilities that have to be included.

**Worksheet 8**

**MIXED PROBLEMS**

This page could be used as a form of assessment, as different problem types are included. There are no scaffolds. Solutions would demonstrate the level of understanding which has been mastered.

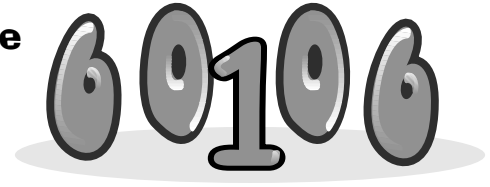


Name \_\_\_\_\_

Date \_\_\_\_\_

# Palindromes

**Palindromic numbers are numbers which are the same when read backwards or forwards.**



- 1 Write the palindromic numbers between 200 and 300.

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- 2 Write the four-digit palindromic numbers between 9 000 and 10 000.

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---

- 3 Write two five-letter palindromic words. Use the letters E, V, L for the first word and M, D, A to make the other.

---

- 4 Write all the three-digit palindromes which have 6 at the centre.

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- 5 Write the palindromic dates which occurred in 2002 when they were written in numeric form, eg 15 February 2008 would be written 15 – 2 – 08 (not palindromic).

---



---

- 6 How many palindromic house numbers in the 400s can you make? Write them down.

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- 7 Write six sets of two palindromic numbers which will total 585?

---



---

- 8 Which sets of palindromic numbers less than 1 000 have a difference of 727?

---



---



Name \_\_\_\_\_

Date \_\_\_\_\_

# Aging families

1 Sisters Bonnie and Blondie are in their twenties and their ages are two years apart. The total of their ages is 50. In how many years will the total of their ages be 60?

**Solution**



Total of ages	Ages of sisters
50	_____ + _____
	_____ + _____

Benny's age	Brother's age	Father's age

2 Benny's father is 30 years old. He is three times Benny's age and eighteen years older than Benny's brother. In how many years will the sum of Benny's age and his brother's age equal their father's age?

**Solution**



3 Little Jim is 5 years old and his mother's age, 30, is the product of his age and his sister's. In how many years will this product be three times what it is now?

**Solution**



Jim's age	Mother's age	Daisy's age



Name \_\_\_\_\_

Date \_\_\_\_\_

# Paying the toll

## Work systematically!

1 To pay a toll of £5.50 on the M6 I cannot use 5p, 2p or 1p pieces. List the ways I can use notes or coins to pay the toll exactly, never using more than 2 of any one coin.

**HINT** List pounds, then pence in each combination.

Notes and coins	Total

Notes and coins	Total

2 If I paid the £5.50 toll with a £10 note, how many different ways could I receive change using nothing smaller than a 50 pence piece? Write the list.

**Solution**

3 So we can pay road tolls easily, we save coins. Every week I save one 50p, one 20p and one 10p piece. My cousin, Nicky, saves one 50p. In how many weeks will I have saved £4 more than my cousin?

**Solution**

Week	My savings	Snippy's savings
1		
2		



Name \_\_\_\_\_

Date \_\_\_\_\_

# Marketing madness

1 The Bling Blang Bloon Company wants to sell more balloons so prize packs are made. On every 3rd packet a bronze seal can be found, on every 4th packet a silver seal can be found and on every 5th packet a gold seal appears. There is a major prize for the person who finds the packet with all three seals on it. How many would have to be made before this lucky packet appears? Write the multiples down the list to see where they all meet.

3s	4s	5s

**Solution**



Red	Blue	Yellow	Purple

2 The Bling Blang Bloons are sold in packs of 10 and there are 4 different colours in a pack. There have to be 3 red Bloons and 2 blue Bloons in a pack but any number of yellows and purples. How many different combinations of the four colours can there be?



Make an organised list to show the different combinations.

**Solution**

3 At Dizzy Dan's Discount Store the Product Placement Department has a major problem with the placement of the latest range of Dizzy's Dishwashers. The Swisher, the Sloser, the Blitzer and the Ritzer are the models to be displayed in a line. The Sloser, however, cannot be placed next to the Ritzer, as it looks too inferior.



1st	2nd	3rd	4th

**Solution**

How many ways can the Placement Department arrange the dishwashers for display?



Name \_\_\_\_\_

Date \_\_\_\_\_

## Fitness fanatics

- 1 In the Skip Daloo Competition, Francie wants to incorporate Jog Jump, Cross-over, Pepper and Heel-toe in her routine. In how many different ways can she arrange 10 skips of these four different skipping styles if she always begins with Jog Jump?

**Solution**



- 2 Francie's classmate, Jazzo, is making up an aerobics pattern to try on his friends. He wants to include 2 lots of stepping, 2 lots of star jumps and a session each of side steps, knees-up and skiing. How many different routines that begin and end in stepping can he make up, star jumps being 3rd and 5th in the routine?

**Solution**



- 3 In a tennis competition with 4 players, each player must umpire one or more matches. Make up a draw for Round 1 for Guy, Hong, Fabia and Mario to play each other once and state who will umpire each match. Use three columns headed Player 1, Player 2, Umpire. Write an observation about the umpiring duties when you have finished.





Name \_\_\_\_\_

Date \_\_\_\_\_

## Timetables

- 1 Jeremy has homework every week in 6 different subjects; Maths, Spelling, Reading, Writing, History and Science. He has 4 nights to do the homework and 3 lots each of Maths and Reading, 2 lots each of Spelling and Writing and 1 lot each of History and Science. How can the timetable work so that he does different subjects each night?



- 2 Mrs Swishenswipe cleans the Brandishes home 5 days a week. She cleans the bathrooms and bedrooms every day, the kitchen twice a week, the dining and living rooms once only in the week. There are 2 bathrooms and 3 bedrooms. Once a week she sweeps the patio. Make a schedule for her to do 6 tasks a day and get all of them done the correct number of times.





Name \_\_\_\_\_

Date \_\_\_\_\_

## For the tummy

1 At Pete's Pizza Place, there is a vast array of choices for toppings. You may choose from Verry Veggie or Marvo Meat as a base, but you may order more cheese, tomato or seafood on top, or any two of these toppings as well.



Six friends walk into the Pizza Place and think they will all have something different to each other so they can share.

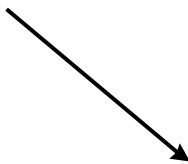
a Will they be able to order all possibilities if they order 1 pizza each? \_\_\_\_\_

b How many choices are there? \_\_\_\_\_

Verry Veggie



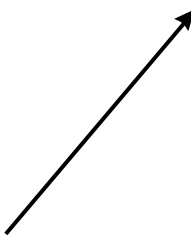
Marvo Meat



2 Two families are going on a picnic. They decide to provide the food between them. The Davidos will take the meat and bread and the Cubbies will take tabouli, cucumber, tomato and lettuce. How many different sandwiches with at least two fillings, can they make with these ingredients?



Sandwich







Name

Date

# Mixed problems

1 How many ways can you arrange the digits 9, 8, 7, 6, to make numbers less than 9 000?

**Solution**

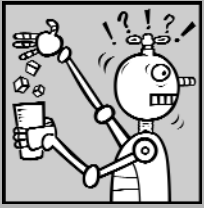
2 The sum of my mother's and my father's ages is 70 and my father is two years older than my mother. With my age added, our total is 80. How old will we all be when our total age is 101?

**Solution**



3 There are 4 teachers at Zander Primary School (Mr J, Miss K, Mrs T, Mr P) and they must share the playground duties equally. There are 4 duties a day — *Before school, Break, 1st Lunch, 2nd Lunch* — for the 5 days of the week. The teachers do not like having the same duty two days in a row. Mr J doesn't like *Before school* on Mondays and Miss K doesn't like *2nd Lunch* on Fridays. Make a Duty Roster so that the teachers will be happy. Work systematically.

Duty:	Before school	Break	1st Lunch	2nd Lunch
Monday				
Tuesday				
Wednesday				
Thursday				
Friday				



# Work backwards

## Rationale

This method is useful when the end result is known, but the initial or intermediate data is not known.

## Teaching *Work backwards*

Carefully read all the information given. Determine what is known and what is not known. Highlight the piece of information which is known – a definite answer. Students should be able to see if the final result will be *more or less* than the piece of information that is given. Knowledge of inverse operations is necessary, as the term *working backwards* implies working in the opposite direction. The following understandings are required:

**A** Strategy for using *Work backwards*

**B** Main skills

### A Strategy

- 1 Discuss which data is the answer at the end – the final result.
- 2 Discuss and highlight the data which has been found.
- 3 Use diagrams or equations to set out working.
- 4 Have students explain their answer and how their solution is proven correct by application of the original data.

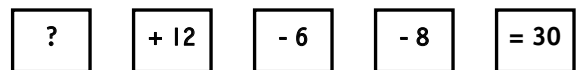
### B Main skills

- 1 **Determining which data is the given or certain piece of information**
- 2 **Determining which is the unknown piece of information**
- 3 **Determining other data to be used**
- 4 **Estimation**

Decide whether the answer will be larger or smaller than the given information.

### 5 Using the diagram method

The working is drawn up using the pieces of data and the signs of the processes, eg addition or subtraction.



### 6 Using the equation method

Write an equation from the given data, using a letter or a symbol for the unknown.

### 7 Using inverse operations

Solve the equation by writing its inverse.

$$U + 19 - 6 \times 3 = 1; \text{ Therefore } U = 1 + 18 - 19; U = 0$$

### 8 Check

Test the solution against the data in the problem by reading the problem and substituting data for the unknowns initially used.

### 9 Explain

Have students explain why their calculation is correct. How does it meet the criteria in the original problem?



**Worksheet 1**

**MY FAVOURITE NUMBERS**

This page provides practice at using inverse operations and using an equation.

Using a symbol for the favourite number, students write out the rest of the calculation, then rewrite it using inverse operations. They must use a symbol each time 'my favourite number' is mentioned.

**Worksheet 2**

**MONEY IN THE BANK**

The unknown is mentioned first, so a symbol is to be chosen for that. Make sure the unknown is clearly understood so that students know what it is that they are finding, and that this is the first amount in the equation.

**Worksheet 3**

**HIKING CHALLENGES**

This is a similar activity to the last two work sheets. Drawing a time line is a way for students to demonstrate that they truly understand the problem and its solution.

**Worksheet 4**

**SHOPPING SPREE**

Firstly, students must find the item that has the given price. Write it down, then list the other items in reverse order. Make the calculations according to the directions in the text.

**Worksheet 5**

**FEARLESS WITH FRACTIONS**

Knowledge of fractions comes into these equations. Students must understand that 1 whole is the 'whole job' or 'whole project' and that parts of it are completed, leaving a part not done.

**Worksheet 6**

**DART GAMES**

Start at the end to find a final score, then work backwards to fill in all the other pieces of the puzzle. Don't forget to use the inverse operations, eg *4 more than*, will mean *subtracting 4* to obtain the result.

**Worksheet 7**

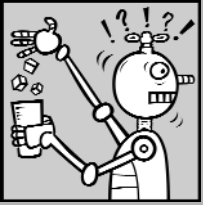
**FOR INTREPID TRAVELLERS**

Look for the final result at the end. Write the list in reverse order. Calculate accordingly.

**Worksheet 8**

**RATES AND CHARGES**

Discuss bills and how they are constructed. Using rates and charges for different parts of a bill, students are gaining experience at real-life situations. They learn to subtract the initial charge, (eg day rate) then divide the remainder to find the pro rata charges.



Name \_\_\_\_\_

Date \_\_\_\_\_

# My favourite numbers

**Write a number sentence for each problem. Use inverse operations to find the answer.**

Example

If you add 10 to my favourite number you have a number that is the square of half a dozen.

Number sentence:  $\blacktriangle + 10 = 6 \times 6$  Take 10 away (inverse operation) from 36

( $\blacktriangle = 6 \times 6 - 10$ ), to arrive at the favourite number, 26.

- 1** If you take my favourite number, multiply it by the sum of 7 and 9, and take away 14 you arrive at the answer of 18.

Number sentences \_\_\_\_\_

**Solution**

- 2** After you have divided my number by 9, added 14 and multiplied by 3, you will end up with 66.

Number sentences \_\_\_\_\_

**Solution**

- 3** If you halve my number, square the answer and subtract the difference between 70 and 36, you will arrive at a number equivalent to 11 times 6.

Number sentences \_\_\_\_\_

**Solution**

- 4** If you double my number and add 4 you arrive at a total that is 7 times 3 less than the square of 11.

Number sentences \_\_\_\_\_

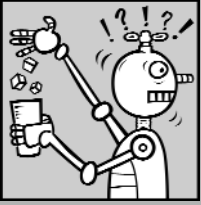
**Solution**

## Challenge

- 5** When you triple my number, add the multiple of 4 and 5 and take away the prime number before 40, you end with 2 less than 5 dozen.

Number sentences \_\_\_\_\_

**Solution**



Name \_\_\_\_\_

Date \_\_\_\_\_

## Money in the bank

- 1 I can't remember how much I had in my purse before I was paid £500 for work. Then I paid the phone bill of £150. I also paid the rent of £820 and still had £130 left. How much was in my purse at the beginning? \_\_\_\_\_



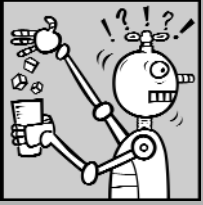
- 2 My bank balance has been erased from the statement. I can see that I paid £280 for car repairs and received £320 from my brother's business. After that, I had two amounts of £60 paid into my account before I paid £750 to the insurance company. There are now £60 in my bank. What was my bank balance at the beginning? \_\_\_\_\_

- 3 My school raised a large amount to send to the Klever Kids Charity. We receive a large donation of £700 more. We have to pay £200 for the advertising and £350 for the rental of the Funny Castle used in the fundraising. With £50 more, we will reach our goal of £1 500. How much did we raise in the first place? \_\_\_\_\_



### Challenge

- 4 Grandpa gave Jock one note with which to pay for his day at the beach. His little sister gave him £5.50 for his train fare, but it only cost him £3.30. He spent £7.20 for lunch and then had £5 left to share with his friends. How much did he get from Grandpa? \_\_\_\_\_



Name \_\_\_\_\_

Date \_\_\_\_\_

## Hiking challenges

- 1 Danny Boy took up the challenge to walk from Glendale to Castle Tops, travelling through Barnside. It took him 1 hour 30 minutes to walk from Glendale to Barnside. After a rest of 25 minutes, he walked for 55 minutes and arrived at Castle Tops at 3:30 pm.

When did he leave Glendale? \_\_\_\_\_

**Draw a timeline to illustrate your solution**

- 2 The scout troop accepted the challenge to walk, with full backpacks weighing 15 kg each, from Springdale to Riverdale. They took 2 hours 5 minutes to get to Clearview, then they took another 50 minutes to reach Riverdale, arriving at 4:30 pm.

When did they leave Springdale? \_\_\_\_\_

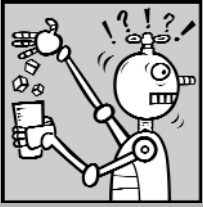


**Draw a timeline to illustrate your solution**

- 3 The guide group tried to beat the scout troop and set out to walk from Debdale to Craghead, a distance of 40 km. They took 2 hours 25 minutes to reach Changers. Then, walking from Changers to Craghead, they took a further 45 minutes.

a If they arrived at Craghead at 5 pm, when did they leave Debdale? \_\_\_\_\_

b How much longer did they walk than the scout troop? \_\_\_\_\_



Name \_\_\_\_\_

Date \_\_\_\_\_

# Shopping spree

**Meena, Anthony and Alexander are shopping for the holidays. They love talking in riddles, however, so it is very hard to tell exactly what they spent!**

**1** Meena bought a top, a skirt, a belt and some sandals. The top was £20 less than the sandals, while the skirt cost £6 more than that. The belt was a bargain at £20 less than the skirt.

If the sandals cost £49, how much did Meena spend on each item and in total?



Top \_\_\_\_\_ Skirt \_\_\_\_\_ Belt \_\_\_\_\_ Sandals \_\_\_\_\_ Total \_\_\_\_\_

**2** Anthony wanted to be outfitted correctly for the new Soccer season. He went to buy boots, shorts, shirt and socks. His shorts were £10 less than the shirt, which was £20 less than the boots. His socks cost the least, a full £19 less than his shorts.

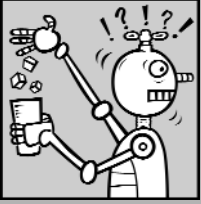
If his boots were £65, how much was each item and what was the total spent?



Shirt \_\_\_\_\_ Shorts \_\_\_\_\_ Socks \_\_\_\_\_ Boots \_\_\_\_\_ Total \_\_\_\_\_

**3** Alexander went to his ATM and drew out a large amount of money for his expenses. For Christmas presents for his family, he spent £15 more on books than on CDs, which cost £15 more than toys. Cards were one-fifth the cost of toys and one-third the cost of calendars, which were £36. Alexander now had £27 left.

How much did he get from the ATM? \_\_\_\_\_



Name \_\_\_\_\_

Date \_\_\_\_\_

## Fearless with fractions

**Calculate what fraction remains, then give it a value.**

- 1 a At the Car Wash fundraising last Saturday, The Angels Team washed  $\frac{1}{2}$  the total cars, The Bizzies Team washed  $\frac{1}{4}$  of the total and The Friskys Team washed the last 10 cars.

How many cars did The Angels Team and The Bizzies Team each wash?

\_\_\_\_\_



- b Draw a diagram illustrating the fractional parts and numbers of cars to check your solution.

- 2 a When the house had to be painted, the Manoogle family invited their uncles to help out. Uncle Val painted for  $\frac{1}{3}$  the total number of hours needed for the whole job, Uncle Joe painted  $\frac{1}{4}$  of the total hours and Old Uncle Freddie managed  $\frac{1}{6}$  of the hours. There were 15 hours of work left for Uncle Ted to complete the job.

How many hours did each Uncle work? \_\_\_\_\_

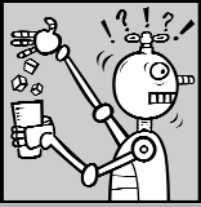


- b Draw a diagram to show your solution.

- 3 In the Chompy Chocolate Sale at their school, Class 5J sold 20%, 5S sold 30%, 6K sold 25%, and 10% was sold by 6C. The rest were sold by 6M for £450.

How much money did 5J and 5S each make? \_\_\_\_\_





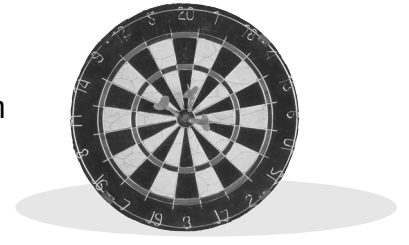
Name \_\_\_\_\_

Date \_\_\_\_\_

# Dart games

**As you read the problem, write down the working in the form of a number sentence, using a symbol for the unknown number at the beginning.**

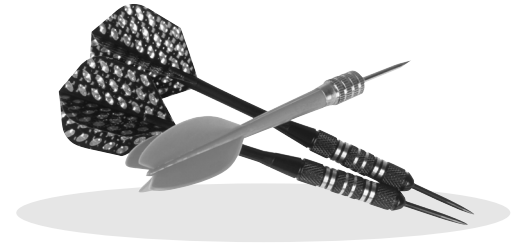
**1** The big, final darts game got under way right on time. Jack threw and amazingly scored an all-time best, beating his old best-ever score by 3. Then Jock stepped up and beat that score by 5. Jack was angry and, with a grim look on his face, bettered Jock's score by another 5 points. Jock didn't like that, so he chose his favourite, true-flying dart and with it, scored 99, 2 better than Jack's previous throw. It was all over!



Now work backwards, changing signs. \_\_\_\_\_

By how many had Jock beaten Jack's original 'best-ever score'? \_\_\_\_\_

**2** The Ladies' Darts Game followed the Men's. Florrie sent the first dart flying and scored her best ever by 2 points, but Connie responded with a score 2 better than that. Florrie then scored, beating Connie's by a huge 4 points but Connie came back with a throw 1 better again. Florrie's last throw of 90 failed to beat Connie's by 1, so Connie retained the championship.



Now work backwards, changing signs. \_\_\_\_\_

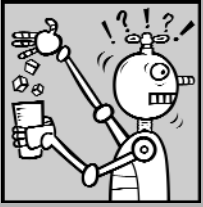
What was Florrie's best ever score before this Championship? \_\_\_\_\_

## Challenge

**3** Connie and Jock, the Voombahs, combined to contest the Club Championship with the Jetskis team. The Voombahs began with an outstanding score, 2 better than their previous record. They then bettered that by 3 points, dropped back by 2 points, and finally finished with a score one better. The Jetskis began their turn with a good throw, bettered it by 2, then another 3 and improved by 2 to end with an 87, 6 better than the Voombahs opening throw.

**a** What were the final total scores? \_\_\_\_\_

**b** What were each team's final throws? \_\_\_\_\_



Name \_\_\_\_\_

Date \_\_\_\_\_

## For intrepid travellers

**1** Mr Mystar planned a very big holiday to celebrate his business success. He Googled air fares for different destinations. Wellington was the cheapest, half the cost of the fare to Shanghai. Hong Kong was £500 less than Shanghai, which was £500 less than Paris, which was £100 more than Tokyo, which was £50 less than London. The fare to London would cost him £2 200.

He eventually decided to book a trip to Shanghai this month and Wellington next month.

**a** What will each fare cost him? \_\_\_\_\_



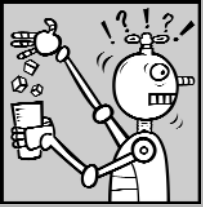
**b** Draw a 'money line' to illustrate your solution. What scale will you use? \_\_\_\_\_

**2** On his Shanghai trip, Mr Mystar decided to backpack along an historic highway to the Great Temple. He begins on a Thursday by travelling 2 hours more than Friday and Saturday combined because of hold-ups along the way. Sunday's travelling time is shorter, 1 hour less than Friday and Saturday combined. On Monday he travels 2 hours more than on Sunday, and on Tuesday the travelling time of 8 hours is equal to Monday's.

**a** How many hours has he travelled in total? \_\_\_\_\_

**b** If he averages 4.5 km per hour walking, how far did he walk during this trip? \_\_\_\_\_

**c** Draw a timeline to illustrate your solution. What scale will you use? \_\_\_\_\_



Name \_\_\_\_\_

Date \_\_\_\_\_

# Rates and charges

**Don't forget to begin with the closing information and work backwards.**

**1** On their annual vacation, Mr Misely decided to rent a car to see the countryside.

**Rental charges:** £82 per day  
55p per kilometre

**a** Mr Misely had to pay £183.75 for the day's rental.

How far did they travel that day? \_\_\_\_\_

**b** If they had gone the extra 52 kilometres to see the Lost Valley of Frish,

how much would the car rental have been? \_\_\_\_\_

**2 Accommodation:** Adults: £55 per night  
Children: £25 per night

Total cost for one night's accommodation = £185

How many children do Mr and Mrs Misely have?

\_\_\_\_\_



**3 Telephone charges:** First minute: 19p  
Each additional minute: 15p

When Mr Misely checked his telephone bill, he thought that his daughter, Chattie, had talked for too long on her calls. How long would Chattie have spoken on the following calls? (HINT: Don't forget to include the first minute.)

**a** Phone number – 8456 3489 Cost £2.74 \_\_\_\_\_

**b** Phone number – 8003 2882 Cost £2.14 \_\_\_\_\_

**c** Phone number – 8987 0021 Cost £4.39 \_\_\_\_\_





# Trial and error

## Rationale

In using *Trial and error* as a problem solving strategy, students must be able to hazard a good guess. This strategy builds a student's understanding and confidence. It gives students a starting point when they can't find any other way to start a solution.

## Teaching *Trial and error*

Use *Trial and error* when there is no starting point in the information given. Students need to be able to make good estimates to use this strategy. They also need to understand how to check against the original information given. The following understandings are necessary:

**A Strategies**

**B Main skills**

### A Strategies

- 1 Carefully read and assimilate the information given in the problem. Discuss which data is to be used when determining a guess. What is a sensible estimate? Will it be more or less than any information in the problem text?
- 2 Write down all the parts of the guess according to the information given.
- 3 Read the problem again, substituting your guesses for the clues. Is it making sense? No!
- 4 Guess again. Adjust your guesses either up or down to fit the clues and reread to check.
- 5 Have students explain their guesses and how their solution is proven to be correct.

### B Main skills

#### 1 Determine which information is to be used

Rule out unnecessary information.

#### 2 Highlight important data

#### 3 Estimate

This is a most important skill. Students must have a feel for where their answers will lie. Large numbers or small? Less than anything in the problem, or more? Ask the students what is reasonable.

#### 4 Check

Test the guess against the information in the problem. If the need is to add the guesses, will they reach the correct total? Checking is most important and must be seen to be a part of the solution.

#### 5 Adjust up or down

When the first guess proves to be incorrect, discuss how to determine whether guesses need to be larger or smaller. What information helps make these decisions? Guide students through the selection of second guesses.

#### 6 Working

Insist that all working is left in place as a record of their thinking. In problem solving exercises it is important to see how the process progressed. Give part marks for incorrect answers if working is in place.

#### 7 Explain

Have students explain how they arrived at their guesses and how they solved the problem.



**Worksheet 1**

**KITCHEN MATHS**

Guessing is the first step. After guessing, they then adjust their answers. Take students through the 'adjusting' step so that they are confident in this.

**Worksheet 2**

**EXTREME SPORTS**

Reading through the instructions carefully is very important. Highlighting the main pieces of data is useful before starting to calculate. Point out which information is not useful to the solution.

**Worksheet 3**

**ADJUST AND READJUST**

Encourage perseverance with guessing. Some students will be able to do this better than others. Encourage reticent students to seek help after they have had a guess. Help them by talking through the parts of their solution that are incorrect and suggest adjustments.

**Worksheet 4**

**SHARE THE WEALTH**

Discuss shares. Show share lists from the papers. The hard part of this exercise is to choose how many of each share. This is a good exercise for an entire term. Use real shares and follow the changes each week. Every week, the students work out the profit and/or loss on their original \$1 000.

**Worksheet 5**

**CRICKET**

Work several demonstration problems following the form suggested at the top of the page. If three numbers are required, start by dividing the total by 3, and then adjust up or down. Discuss how much adjustment needs to occur for certain totals.

**Worksheet 6**

**ROMAN TIMES**

Practise dividing the given total by the number of parts it should have. Read instructions very carefully to be sure that all guides are being followed.

**Worksheet 7**

**GUESS MY NUMBER**

As the students read the problem, demonstrate writing an equation with numbers and signs. Show how to use a letter or symbol for the words 'my number'. eg Q1:  $2 \times N - 7 = N + 5$ . From this, students can guess an appropriate number to begin with.

**Worksheet 8**

**DOOBY ZOO**

Students may be encouraged to make up similar problems for their classmates to solve. Perseverance is important as an incorrect guess will involve several changes in other data.



Name \_\_\_\_\_

Date \_\_\_\_\_

## Kitchen maths

### It's holiday time and Mrs Murphy is baking and cooking for her large family.

- 1 Mrs Murphy has 4 cups of flour in the pantry. The recipe for the cake needs  $\frac{1}{4}$  cup more than the pies and she will completely use up the 4 cups. How much flour is required for both recipes?



Guess	Check
Guess	Check
Guess	Check

Solution \_\_\_\_\_

- 2 When Mrs Murphy cuts three pies, she gives slices to her neighbours. She gives adults twice as much as children. If she gives pie to fewer than 5 adults and to the same number of children, what fraction of a pie will she give to each?



Guess	Check
Guess	Check
Guess	Check

Solution \_\_\_\_\_

- 3 To make a fruit salad Mrs Murphy needs a total of 25 pieces of fruit. She has twice as many kiwi fruit as peaches and the same number of apricots as bananas. There are more strawberries than any other fruit (six times the number of apricots). How many of each does she have?



Guess	Check
Guess	Check
Guess	Check

Solution \_\_\_\_\_



Name \_\_\_\_\_

Date \_\_\_\_\_

## Extreme sports

**The Streme Tracks Comp will decide the champion board, wave and dive contestant of the whole year.**

- 1 In the Board Bash on Wednesday morning, the points were close. Of the total points to be won Big Bolder scored 9s, 8s and some 7s and totalled 73. Jim Jets scored some 9s, 8s and 6s. His total was 77. Flailing Frod scored 9s, 7s and 6s and totalled 77.

What were the three sets of scores? \_\_\_\_\_



- 2 Our three fearless foes were rested and ready to go in the wave riding competition on Thursday morning. The total of the three scores was 225. Flailing is 10 points behind Jim and 20 points behind Big Bolder.

What are their scores? \_\_\_\_\_

- 3 Friday was the dive competition and all were well trained. The dive scores were arrived at by judging style, form and precision. Big Bolder's score was the sum of two primes between 30 and 40, Flailing Frod's score was the multiple of a prime number less than 20, and 5, while Jim Jetski's score was 15 less than a prime number between 80 and 90. The total of the three scores is 201.

What are their scores? \_\_\_\_\_



- 4 Who won the whole Streme Tracks Comp? \_\_\_\_\_



Name \_\_\_\_\_

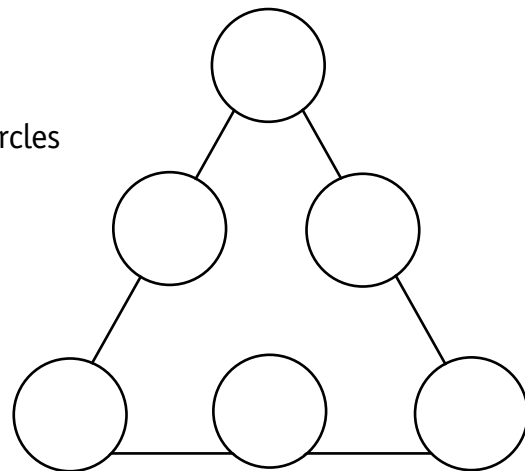
Date \_\_\_\_\_

# Adjust and readjust

1 Using numbers 1 – 9, make the totals of all rows, columns and diagonals 15.


2 Using numbers 1 – 16, make the totals of all rows and columns 34.


3 Using the numbers 1 – 6, make the totals of the circles along each side equal 10.



4 On this balance, make the right side balance the left side using the given masses.

10g, 20g, 30g, 40g, 50g, 60g, 70g, 80g, 90g, 100g, 110g, 120g.







Name

Date

# Share the wealth

**Shares** are a way that people can own part of a business and share in its wealth. When the business makes a profit, the shareholders are given a dividend as a share of the profit. People buy the shares and can sell them when they wish. The Money Makers is a club that invests in shares and all members of the club receive dividends.

The Money Makers had £1 000 and bought shares in 3 businesses. They could buy as many of each as they wanted. Use the table. What shares, and how many of each, could they have bought?

Estimate how many they can afford. Add the costs, then adjust their list until they get a total near to, but less than, £1 000.

It is very exciting to choose real British businesses and follow their shares for some months.

Business	Price
Telgo	£2.25
Big Q	£1.65
Moony	£4.20
Frits Co	£5.60
Postoff	£2.85
Charm	£3.10

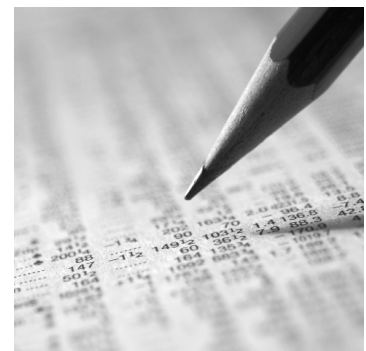
No. of shares	Cost ea.	Business	Cost
			Total =

Adjust

No. of shares	Cost ea.	Business	Cost
			Total =

Adjust

No. of shares	Cost ea.	Business	Cost
			Total =





Name \_\_\_\_\_

Date \_\_\_\_\_

# Cricket

**Hint** – When you have a total to aim for, divide first then adjust up or down.

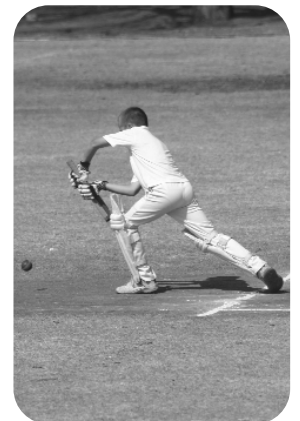
- 1 The Swipems Cricket Club opposed the Blockers Club one Saturday afternoon. Henry, for the Swipems, scored 145, which was as many as Sam, Daniel and Jack together. Sam scored 35 more than Jack and 60 more than Daniel.

What were Sam's, Jack's and Daniel's scores?



## Solution

- 2 At the end of the day, the Smashems had scored a total of 46 more than last week's score and 24 less than the previous week. The total of the three weeks' scores was 350. What are the scores?



## Solution

- 3 The Crookems score was not their best. Their last three weeks' scores totalled 460 with the lowest being 84 less than their highest. What were their last three weeks' scores?

## Solution



Name \_\_\_\_\_

Date \_\_\_\_\_

## Roman times

**Roman centurions commanded troops of 100 soldiers. Their training was very hard and many soldiers perished during their exercises.**

- 1 Each week during training, one more soldier than in the previous week would fail his test (Wk 1 = 1, Wk 2 = 2 etc.).

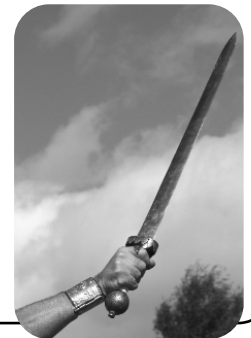
When 72 soldiers were left, how many weeks had they been training? \_\_\_\_\_

- 2 On a very long march, the Romans marched in full battle uniform. They found this very tiring and each day would march 2 kilometres less than the day before. After a few days they had covered 68 km. How many days had they been marching? \_\_\_\_\_



- 3 Last year, Arturio was the champion in swordsmanship. This year, however, he could not improve. His score decreased by between 1 and 3 points every day of the competition. After 3 days, his total score was 269.

What were some scores he could have had? \_\_\_\_\_



### Challenge

- 4 At the end of their training, five troops met to compete. There were a total of 430 soldiers remaining in the troops. In each troop there were between 80 and 90 soldiers. Troop J had an even number, Troop K had a prime number, Troop T had a square number, Troop S had a multiple of 3 and Troop F had a multiple of 5.

How many were in each troop? \_\_\_\_\_



Name \_\_\_\_\_

Date \_\_\_\_\_

## Guess my number

**Numby, the Numba Rhumba, talks in riddles when asked about his numbers. Frustrate him by solving his riddles!**

1 Twice my number minus seven is equal to my number plus five.

**Solution**

2 Three times my number minus eight is equal to my number plus 6.

**Solution**

3 My number multiplied by 5, plus five, is 20 less than eight more than my number multiplied by six.

**Solution**

4 Make up two of your own *Guess my number* puzzles for your classmates to solve.

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Name \_\_\_\_\_

Date \_\_\_\_\_

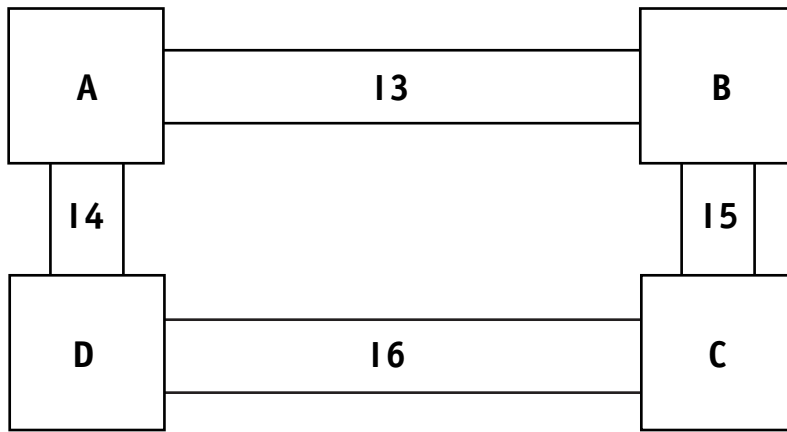
# Dooby zoo

**There has to be a big rebuilding of enclosures in the Dooby Zoo since a hailstorm damaged many of them. After the rebuilding and relocation, Zoo Doobies, the attendants, have to check how many animals are in each enclosure. Please help them check their numbers.**

The numbers on the tunnels are the totals for the two shelters that the tunnel connects.

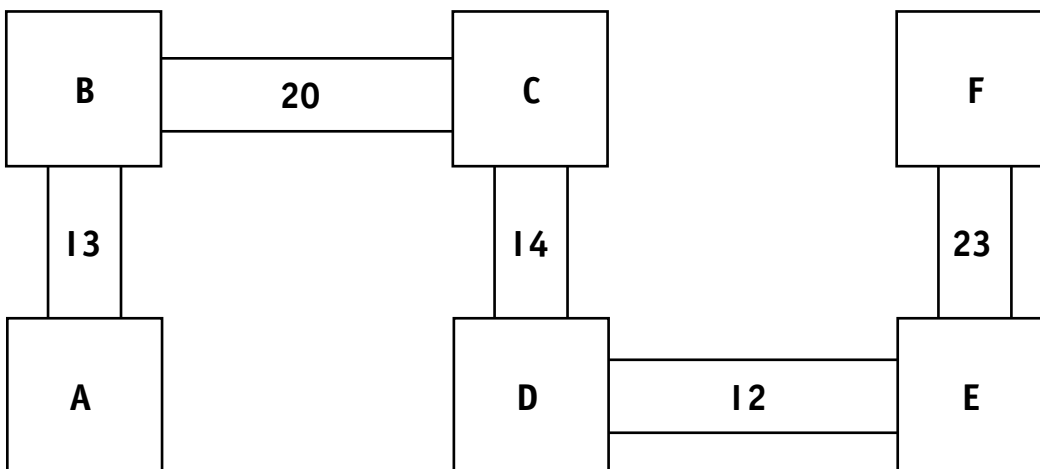
- 1 There are 4 cages of bilbies, who are shy but clever little animals. They won't come out to be counted, but send little messages. The difference between the largest number in a shelter and the smallest is 7.

How many bilbies are in each shelter? \_\_\_\_\_



- 2 In the bush rat cages there are exactly 50 animals in the shelters and none in the tunnels at the time of counting.

Determine how many are in each shelter. \_\_\_\_\_





# Logical reasoning

## Rationale

In using *Logical reasoning* to solve problems, students consider many pieces of data and decide on a systematic method of utilising this data. This involves deciding which piece comes first, *what is not* as well as *what is* and how to set the solution step by step.

## Teaching *Logical reasoning*

The following must be taken into consideration when teaching problem solving through logical reasoning.

- A Different strategies
- B Main skills

### A Different strategies

There are a number of strategies which come under the umbrella of solving problems by *logical reasoning*. Included in these are:

- 1 Draw a grid or matrix, and mark what is a fact and what is not.
- 2 Use a diagram.
- 3 What are the properties of the objects to be considered?

Chiefly the student must understand what they are being asked, what information is to be used and in what order it should be utilised.

### B Main skills

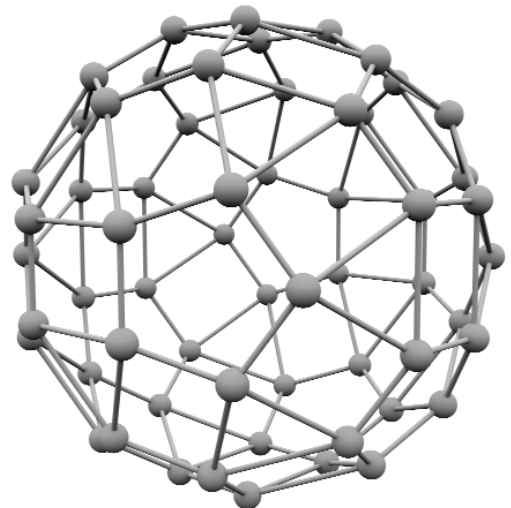
- 1 **Read the problem** with understanding. Highlight the actual question. This is the most important step, as the question and the data often contain tricks or twists that can confuse students.
- 2 **Decide on what data** is to be used. Decide what is known and what needs to be found out. Use a highlighter, leaving out unnecessary words.

### 3 Decide on what strategy is to be used.

- a Draw a grid or matrix and utilise a system to mark *what is* and *what is not*.
- b Use a diagram – position information so that it can be more easily understood.
- c Make a list – simplification of data in the problem so that it can be seen more concisely. Consider all known information about the problem.

### 4 Written work

Apply the strategy. The strategies all require some writing down of data. Insist that students write down their working and can communicate this when asked to.



**Worksheet 1**

**PROPERTIES**

When students understand the concept of *properties*, they have a greater understanding of how groups of objects belong together and what decisions can be made about them. Discuss how we know something is an apple, a triangle, a horse. What are the properties of these objects? Do they remain constant?

**Worksheet 2**

**ODD-MAN-OUT**

This practises applying students' knowledge of properties. They look at a group of objects, see a common thread and apply it to find the misfit.

**Worksheet 3**

**MORE PROPERTIES TO CONSIDER**

This work sheet gives practice at deciding *what is* and *what is not*. Knowledge of various shapes is tested.

**Worksheet 4**

**HOLIDAY TIME**

Careful reading of the problem is necessary so that all data is considered before a Venn diagram is drawn. Numbers are then recorded in the correct sections.

**Worksheet 5**

**STEP BY STEP**

After reading the problem, the student must work through it *step by step*, recording, reading, recording, reading. Make sure that once a piece of data is processed, it is applied to other decisions too.

**Worksheet 6**

**ON BALANCE**

The task is to determine 'if this is one, then this is two,' or similar. There may be more than one step in determining the value of a letter.

**Worksheet 7**

**ONE MOVE AT A TIME**

Perseverance is the main skill to use. Students must develop the skill of *if at first you don't succeed try, try again*. Concentration and recording their moves are also important skills.

**Worksheet 8**

**MIXED PROBLEMS**

This work sheet is a mix of problems all needing a logical approach.



Name \_\_\_\_\_

Date \_\_\_\_\_

# Properties

In these problems you must decide what the properties of the horizontal objects and the vertical objects are. From the given choices, decide on one which will fit *both* the horizontal *and* the vertical groups.

1


- a The horizontals are \_\_\_\_\_  
\_\_\_\_\_
- b The verticals are \_\_\_\_\_  
\_\_\_\_\_
- c My choice is the \_\_\_\_\_  
because \_\_\_\_\_

2

	27	24	54
16			
28	72	44	
56	32	80	

- a The horizontals are \_\_\_\_\_  
\_\_\_\_\_
- b The verticals are \_\_\_\_\_  
\_\_\_\_\_
- c My choice is the \_\_\_\_\_  
because \_\_\_\_\_

3

	64	100	36
72			
120	256	42	
56	96	48	

- a The horizontals are \_\_\_\_\_  
\_\_\_\_\_
- b The verticals are \_\_\_\_\_  
\_\_\_\_\_
- c My choice is the \_\_\_\_\_  
because \_\_\_\_\_

## Challenge

4

	49	81	144
6			
21	36	24	
55	16	66	

- a The horizontals are \_\_\_\_\_  
\_\_\_\_\_
- b The verticals are \_\_\_\_\_  
\_\_\_\_\_
- c My choice is the \_\_\_\_\_  
because \_\_\_\_\_





Name \_\_\_\_\_

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# Odd-man-out

The following sets of objects contain an odd-man-out. Circle the odd one and explain your reason.

1 18, 24, 32, 56, 28, 37, 26

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2 square, octagon, circle, triangle, pentagon

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3 10, 21, 78, 35, 3, 15

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4 75, 63, 27, 12, 35, 42, 81

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5 8, 121, 100, 64, 144, 36

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6 64, 68, 67, 62, 69, 65

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7  $\frac{3}{4}, \frac{5}{6}, \frac{1}{3}, \frac{2}{5}, \frac{7}{12}$

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---

8 10%, 60%, 100%, 80%, 20%, 40%

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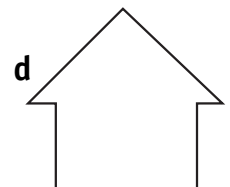
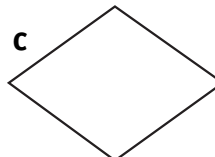
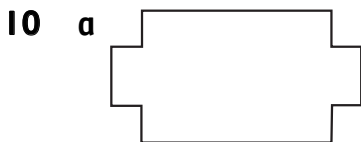
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9 144, 120, 116, 125, 128, 136

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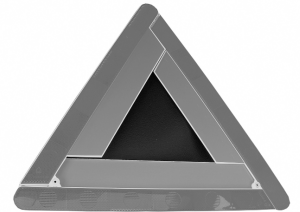
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# More properties to consider

**When you understand the properties of certain objects, you can make judgements about *what is* and *what is not*. Answer true or false to each of the following statements.**

## 1 Triangles

- a All triangles have 3 sides. \_\_\_\_\_
- b There are only two types of triangles. \_\_\_\_\_
- c All equilateral triangles are isosceles triangles. \_\_\_\_\_
- d Some isosceles triangles are right-angled triangles. \_\_\_\_\_
- e Some equilateral triangles are right-angled triangles. \_\_\_\_\_
- f Some isosceles triangles are right-angled and equilateral. \_\_\_\_\_
- g No isosceles triangles are scalene. \_\_\_\_\_
- h Not all isosceles triangles are equilateral or right-angled. \_\_\_\_\_



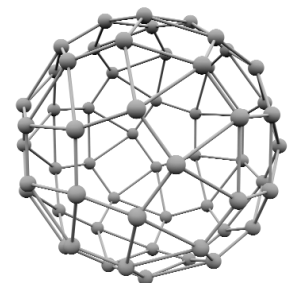
## 2 Quadrilaterals

- a All squares are quadrilaterals. \_\_\_\_\_
- b All squares are rectangles. \_\_\_\_\_
- c All quadrilaterals are squares or rectangles. \_\_\_\_\_
- d All rectangles are parallelograms. \_\_\_\_\_
- e All parallelograms are quadrilaterals. \_\_\_\_\_
- f All quadrilaterals have one right angle. \_\_\_\_\_
- g Some quadrilaterals are trapeziums. \_\_\_\_\_
- h Not all quadrilaterals contain right angles. \_\_\_\_\_



## 3 Polygons

- a All polygons have straight sides. \_\_\_\_\_
- b Some polygons are regular shapes. \_\_\_\_\_
- c Squares and rhombuses are both polygons. \_\_\_\_\_
- d Pentagons all have 5 equal sides. \_\_\_\_\_
- e Some hexagons can have a right angle. \_\_\_\_\_
- f All hexagons have one right angle. \_\_\_\_\_
- g Triangles are not polygons. \_\_\_\_\_
- h Parallelograms are polygons too. \_\_\_\_\_





Name \_\_\_\_\_

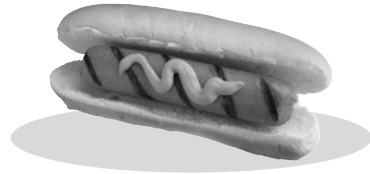
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## Holiday time

**You learned about Venn diagrams in *Draw a diagram* exercises. Organise the following information using Venn diagrams, then answer the questions.**

- 1 At the Chompalot Annual Picnic Day, 65 people ate 40 hot dogs and 35 sausages. Everyone ate at least one hot dog or sausage. Ten people ate one of each.

How many people ate only a sausage? \_\_\_\_\_



- 2 When the Thankful family gathered to celebrate Grandpa's 100th birthday, they ate 12 portions of turkey, 11 portions of ham and 8 portions of chicken. One person ate all three, two people had turkey and ham and 4 people had ham and chicken. Nine people had only turkey, 4 people had only ham.

a How many people had only chicken? \_\_\_\_\_

b How many people were at Grandpa's birthday? \_\_\_\_\_



- 3 Whilst on holidays last summer, Jessie and Jancie decided to write down the numbers used on numberplates on the cars which they passed on the deserted roads. They passed fifteen cars which used numbers 0 to 3, twenty-two cars which used numbers 3 to 6 and twenty cars with numbers 6 to 9. Five cars had 3 on their plates, and six cars had the numeral 6 on their plates.

a How many cars had numbers 4 or 5 on them? \_\_\_\_\_

b How many cars did they pass altogether? \_\_\_\_\_



Name \_\_\_\_\_

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# Step by step

**A matrix is sometimes a good way to solve a problem. We use the process of elimination to arrive at an answer. Use ticks to show 'yes' and crosses to show 'no' as you read the data. As you find a 'yes' you should fill in all the 'no' spaces as well, across and down. eg If Peter has a 'yes' for Red house, then he has 'no' for the other colours and all other names have a 'no' for Red House.**

**1** There are four sports houses in Starlight School — Blue, Red, Green and Yellow. The Captain of Blue House is not Ned, and Ned is not the Captain of Green House. The Captain of Red House is Bernie but Peter is not Captain of Blue House.

	Blue	Red	Green	Yellow
Ned				
Peter				
Barry				
Bernie				

What House is Barry the Captain of? \_\_\_\_\_

**2** Jake has a snake and he is older than Spud and Tito, who is nine. Spud does not have a tortoise. Who is 10 years old? \_\_\_\_\_

	Snake	Mouse	Tortoise	9 yr	10 yr	11 yr
Jake						
Spud						
Tito						

**3** On the soccer team, Fonzie was the Most Improved and played Centre. Danzer does not get the Best Goal Award. The person who is Best and Fairest is a Goalie. Jantzer plays Forward.

Person	Position	Award
Fonzie		
Jantzer		
Danzer		

What position does Danzer play? \_\_\_\_\_

Person	Food	Sport
Davy		
Joe		
Serge		

**4** Davy loves pies best but can't play rugby or cricket, while Joe doesn't like fruit or soccer. Serge plays rugby and can't stand chips.

**a** Who plays cricket? \_\_\_\_\_

**b** Who eats fruit? \_\_\_\_\_





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# On balance

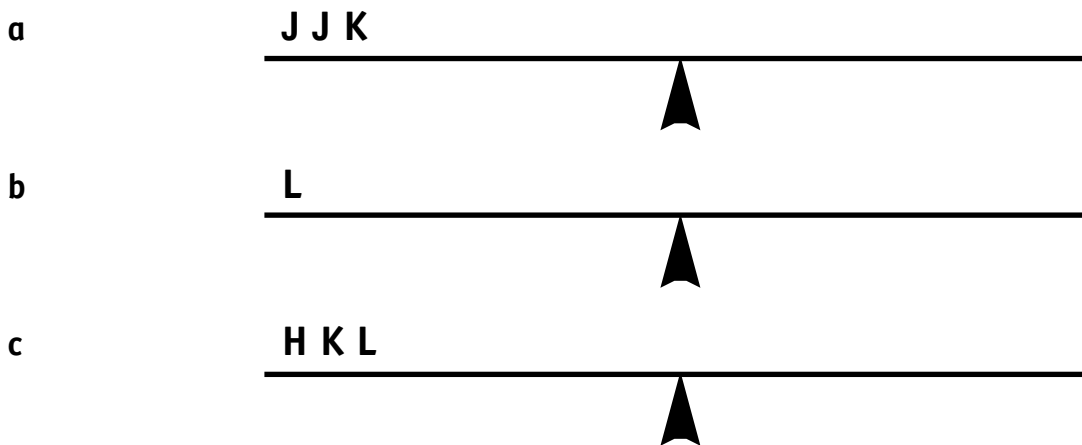
$B = C C$	$C = A A$	$D = C + B$	$E = A + B$
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1 Use only As to balance the scales.



$J = G G$	$H = G K$	$K = J G$	$L = K K$
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2 Use only Gs to balance the scales.





Name \_\_\_\_\_

Date \_\_\_\_\_

## One move at a time

- 1 Use two blue counters and two red counters on the grid below. How many moves will it take you to change the places of the red to the blue and the blue to the red?

Blue must always move towards the red end. Red must always move towards the blue end.  
The counters may only jump over one counter at a time.

a Record your moves in this manner.

**Begin** R R – B B; **1st move** R R B – B; **2nd move** R – B R B; **3rd move** \_\_\_\_\_

red	red		blue	blue
-----	-----	--	------	------

b Number of moves \_\_\_\_\_

- 2 Use three red counters and three blue counters. How many moves are necessary to have them change places? Is there a pattern to the successful moves?

a Record your moves on spare paper.

red	red	red		blue	blue	blue
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b Number of moves \_\_\_\_\_

c Describe the pattern in the moves.

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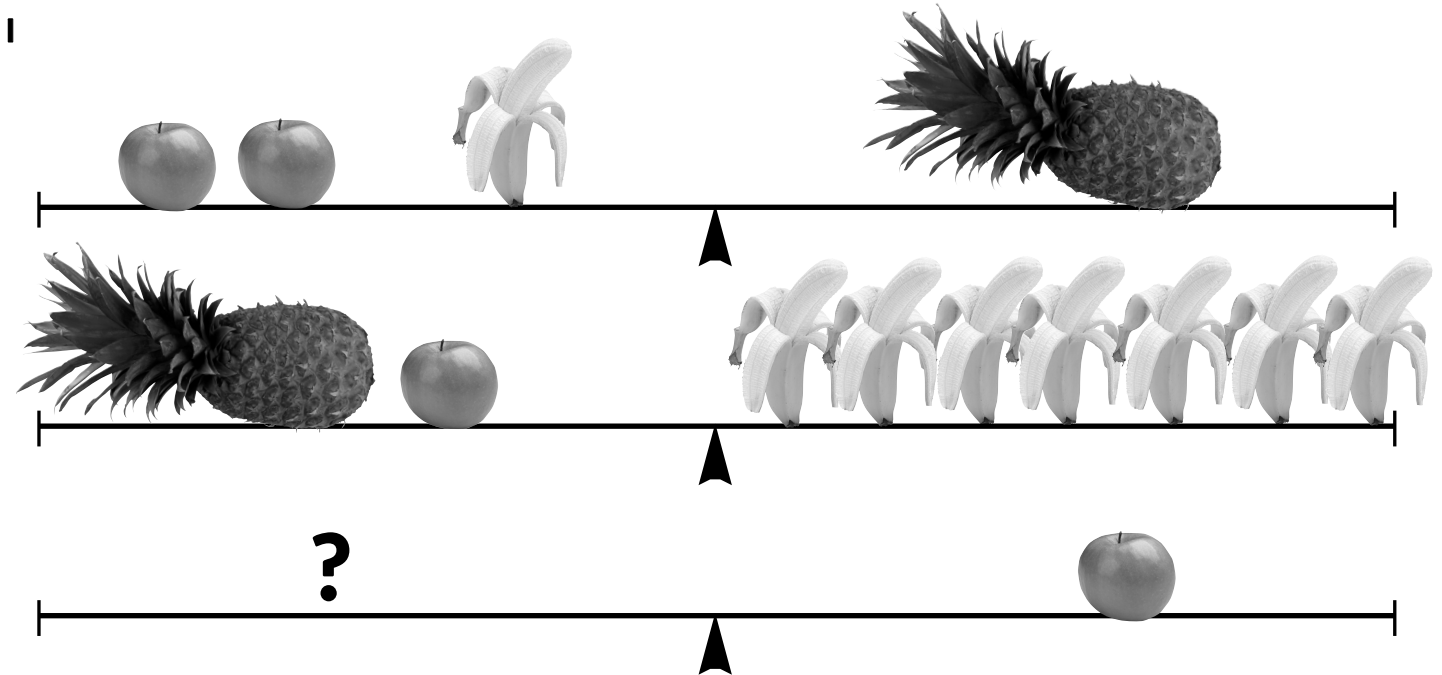
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# Mixed problems



How many bananas are needed to balance the apple? \_\_\_\_\_

2 Cut out 6 triangles and place them in the positions of Diagram 1. Change Diagram 1 to Diagram 2 by turning pairs of triangles next to each other upside down. Do it in the least number of turns you can.

Diagram 1



Diagram 2



3 Four ladies are going to a Gala Lunch and each one needs something new.

Name	Hat	Dress	Shoes	Handbag

One buys a new hat, one a new dress, one buys new shoes and the other a new handbag for the day.

Maggie already has a dress, and hat. Mandy buys a new handbag to go with her old shoes. Mimi does not get a hat or shoes. Molly chooses a hat she cannot live without.

Who buys new shoes? \_\_\_\_\_



# Open-ended problem solving

## Rationale

*Open-ended problem solving* gives students a great opportunity to be creative in a mathematical sense. They can play with solutions, experiment, ask questions and see the challenge in solving problems without one set answer. For the divergent thinker, there is the possibility to be individual in finding a solution and the ability to take additional time researching a possible solution.

.....

## Teaching *Open-ended problem solving*

The following should be considered in the teaching of problem solving where there is no one particular correct solution.

**A** Thinking logically

**B** Persevering to find alternative solutions

### A Thinking logically

- 1 Understanding the problem. Students must have good understanding of what diversity is possible in the solution.
- 2 Possibilities of the solution. Students must have some idea of what is the possible solution, and how that can be arrived at.
- 3 Progressing through solutions in a logical way.

### B Persevering to find alternative solutions

**I** Many solutions are possible

Students who succeed at open-ended problem solving will love to keep looking for novel and unusual solutions. They must not wish to merely put down one answer and move on. They will offer many solutions and test their validity.

**2 Rearrange solutions** so that another one appears.

**3 Use concrete materials** such as counters etc. to take the place of objects when moving them around to find alternate solutions.

**4 Take as much time as necessary** to reach a thorough solution and understanding.

**5 Reflect upon the solution and explain it to others**

The ability to verbalise a solution and relate the explanation to the original problem shows full understanding and mathematical thinking.





**Worksheet 1**

**TAKE THE CHALLENGE**

Encourage students to work systematically through the number sentences, increasing one digit by 1, decreasing another by 1 etc. to move through all possibilities in an organised way. They decide whether they need to go higher or lower to help plan the next answer.

**Worksheet 3**

**ARTY MARTY**

Allow extra paper for students to develop different solutions for this problem. Hold a class competition and work on this in spare time as well. Put completed solutions on the board for all to share.

**Worksheet 5**

**THE STORY OF A GRAPH**

Every graph tells a story. Allow students to be creative here as long as the narrative matches the data on the graph. Make sure they understand what the data is showing before they begin their stories. Share stories. The rest of the class follows the graph to check.

**Worksheet 7**

**SHOP FOR THE TOP**

Children can choose their favourite foods. There are many choices and no one answer is correct. Students will also see how a shopping list adds to a large amount very quickly. Discuss family budgeting for weekly expenses.

**Worksheet 2**

**DIFFERENT PLANS**

Students think about the factors of 24. Rooms that are irregular shapes are permissible so L-shaped rooms or even triangular shapes may be possible for high-ability students.

**Worksheet 4**

**MISSING NUMBERS**

Room for creativity in this page! Allow students to express their unusual ideas about numbers. Make sure they are not too repetitive. Reward originality.

**Worksheet 6**

**FIVE-MINUTE CHALLENGES**

Timing the students at this task will encourage them to work smart. Record interesting equations on a chart for students to check and use as motivation.

**Worksheet 8**

**SOME REALLY BIG NUMBERS**

Students have the opportunity to work freely and with a partner to discuss working and what answers make sense. Make sure they know how to check what a reasonable answer is. Remember, open-ended problem solving will take longer and much learning about strategies and good answers will take place with open-ended time as well.



Name \_\_\_\_\_

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## Take the challenge

**Allow yourself 5 minutes for each question.**

**You may need a spare piece of paper if you fill these spaces!**

20 = Excellent	15 = Very Good	10 = Good
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**1** Make as many number sentences as you can with an answer of 10.

Use only addition and subtraction and single-digit numbers.

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**2** Make as many number sentences as you can with an answer of 16.

Use addition, subtraction and multiplication and single-digit numbers only.

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**3** Make as many number sentences as you can with an answer of 20.

Use addition, subtraction, multiplication and division and single-digit numbers only.

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**4** Make as many number sentences as you can with an answer of 24.

Use the four processes and fractions as well as single-digit numbers.

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Name \_\_\_\_\_

Date \_\_\_\_\_

# Different plans

**Use the squared paper (scale 0.5 cm = 1 m) to lay out your plans for the following questions.**

**1** You have 24 m<sup>2</sup> of carpet. What sized rooms can you carpet with it? The rooms can be regular or irregular shapes.

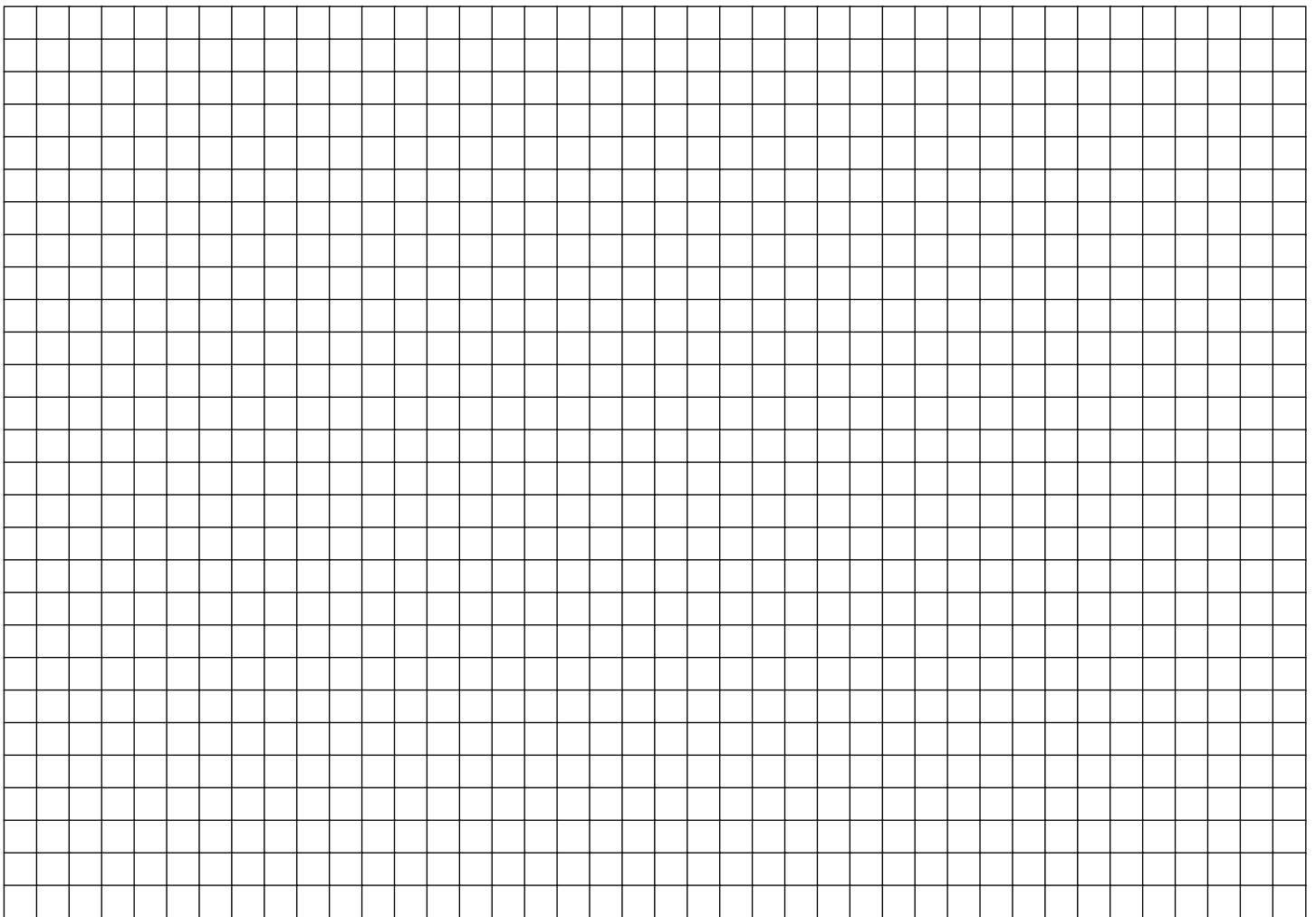
Are the rooms all the same size in area? \_\_\_\_\_

Do they all have the same perimeter? \_\_\_\_\_

**2** You have 30 m of rope. What sized sections can you rope off on your lawn? They may be regular or irregular shapes. Use thread 15 cm long for the rope and pins for the corners.

Do the shapes have the same area? \_\_\_\_\_

Do they have the same perimeter? \_\_\_\_\_



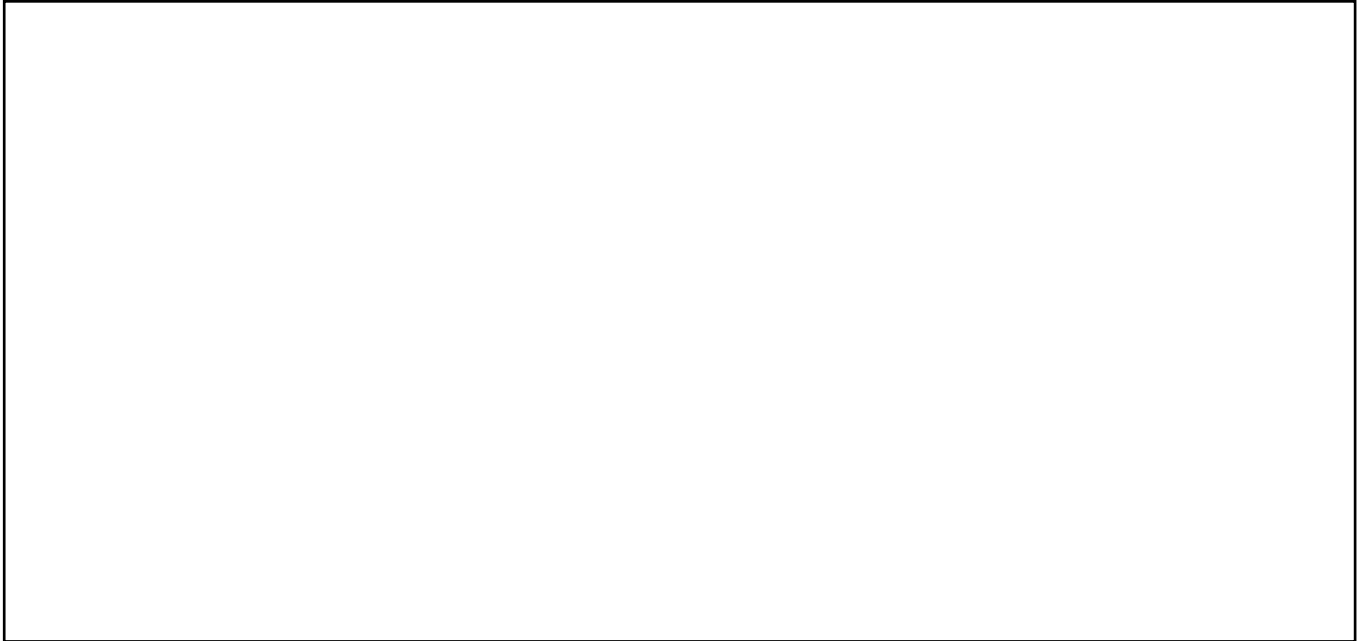


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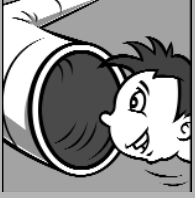
## Arty Marty

- 1** In the rectangle, draw lines from one side to another side to make 12 triangles. What is the least number of straight lines that you can draw to complete this challenge? Carry on the challenge on another sheet of paper.



- 2** In this rectangle, draw lines from one side to another side to make 12 rectangles or squares. What is the least number of straight lines that you can draw to complete this challenge? Carry on and try other methods on spare paper.





Name \_\_\_\_\_

Date \_\_\_\_\_

# Missing numbers

The following numbers have escaped from the Numerocells where they were kept for multiple crimes of fraud. The whole Digiforce is out to bring them in but descriptions are needed to assist in identification.

Make a poster for each missing number, describing its qualities, eg a multiple of 7, a very odd number with two digits, a prime 'suspect' between 25 and 30, etc.

1

**WANTED! 72**

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2

**WANTED! 84**

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3

**WANTED! 96**

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4

**WANTED! 42**

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Name \_\_\_\_\_

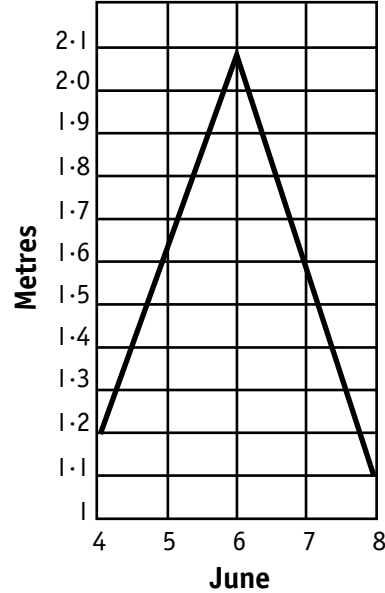
Date \_\_\_\_\_

# The story of a graph

**This graph tells the story of a river in flood.**

At midday on 4 June the water height above the bridge deck was 1.2 m. By midday 6 June, the water had reached a peak of 2.1 m above the bridge deck. For the next two days the river level dropped and was no longer a threat to the people living nearby.

**Flood height**



**Tell a story which could match the data in these graphs. Make sure you put in the necessary labels!**

1

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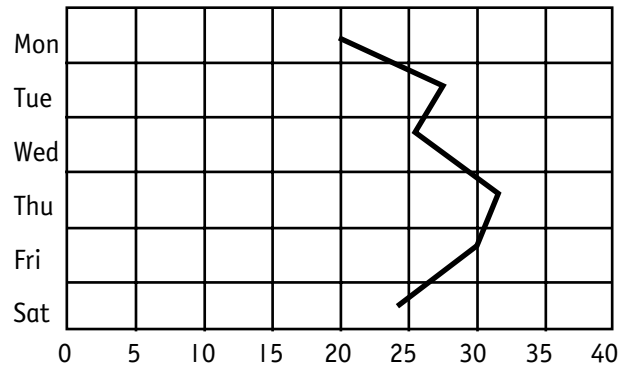
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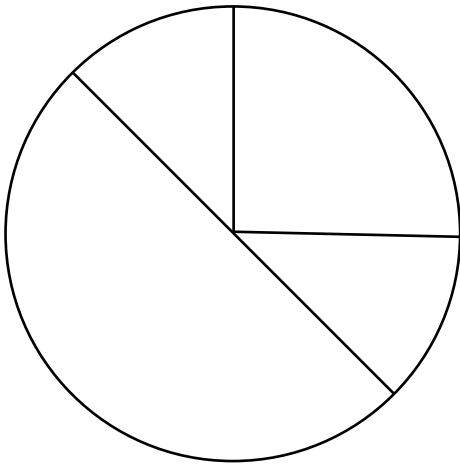
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Name \_\_\_\_\_

Date \_\_\_\_\_

## Five-minute challenges

**You are allowed just 5 minutes for each of these challenges.  
You may need spare paper.**

- 1** Use the signs + and - to create number sentences equalling between 10 and 20. Use only the digits 2, 4, 10, 12, 14. They do not all have to be used every time, but can only be used once in each number sentence.

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- 2** Use the signs +, -,  $\times$  or  $\div$  to create number sentences equalling less than 20. Use the digits 2, 3, 4 and 5. They do not all have to be used every time, but can only be used once in each number sentence.

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- 3** Use the signs +, -,  $\times$  or  $\div$  to create number sentences equalling between 30 and 40. Use the digits 5, 6, 8 and 10. They do not all have to be used every time, but can only be used once in each number sentence.

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- 4** Use brackets, +, -,  $\times$  or  $\div$  to create number sentences to equal exactly 100. Use all the digits 2, 4, 5, and 0 only once each in each number sentence.

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- 5** Use any combination of the digits 2, 4, 6 and 8 with any signs to equal;

**a** 1 \_\_\_\_\_

**b** 2 \_\_\_\_\_

**c** 50 \_\_\_\_\_



Name \_\_\_\_\_

Date \_\_\_\_\_

# Shop for the top

1 With £100 to spend, Bobbie and Barbie Brand goes out to do their weekly food shopping. They always make sure they have plenty of fruit and vegetables, they eat no bread and prefer chicken to steak. They enjoy cheese and crackers as well. Their children like Fruit'n'jelly snacks, Nuts-for-Nuts cereal with slim milk, fish and Choc Chipper Cookies.

What items could they take home for a total as close to £100 as possible?  
They may choose multiples of the same item to make up the total.

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Apples	£4 kg	<b>Choc-Chipper</b>
Bananas	£8 kg	Cookies
Grapes	£9 kg	£3.85 pack
Pineapple	£2.50 ea	Cheese
Broccoli	£5 kg	£4 pack
Bread	£3.25 loaf	Crackers
Chicken	£6.50 kg	£3 pack
Steak	£11.00 kg	
Fish	£15 kg	
Fruit'n'Jelly	£1.25 ea	
Cereal	£4.75 box	
Slim milk	£2.85 L	



2 Marcus Maximum went out to fill his shopping basket and had just £50 to spend on his weekly list. He enjoys cheese and crackers, chicken, fish, bread, all sorts of fruits and vegetables, and takes slim milk in his tea. What could he have bought to spend close to his £50?

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Name \_\_\_\_\_

Date \_\_\_\_\_

## Some really big numbers

### 1 How many breaths do we take in a lifetime?

Working with a partner, count your breaths in one minute. Work out how many that is in a day, a year, a lifetime. Show your working and check it to be sure you have followed through with all the steps.

How long do you think you might live? \_\_\_\_\_

Is there a correct answer to this problem? \_\_\_\_\_ Explain. \_\_\_\_\_

### 2 How many words can you read in a minute?

Working with a partner, read a page of a book which you can read fluently and with expression. Read for five minutes exactly. Mark the last word with your finger. Count the words read. Divide by 5.

How many words would you read in an hour? \_\_\_\_\_

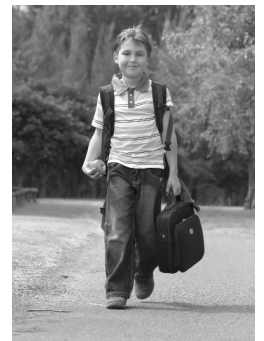
How many in 3 hours 15 minutes? \_\_\_\_\_



### 3 How many steps do you take in a day? You can wear a pedometer to count your steps or you can work out how many you think you take. Work outside and count the steps you take walking to various places around the school, eg library, canteen etc. Estimate how many steps you will take getting home from school in the afternoon.

What will be your goal in the number of steps per day? \_\_\_\_\_

What will you have to do to make that many steps? \_\_\_\_\_



Our Prime Minister regularly walked 10 000 steps per day in 2006.